NEW PERSPECTIVE OF FRACTURE MECHANICS INSPIRED BY NOVEL TEST WITH CRACK-PARALLEL COMPRESSION

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Video link: https://youtu.be/UyTeopPModA
Standard Fracture Test Specimens — negligible crack-parallel normal stress
Why has the crack-parallel stress $\sigma_{xx}$ been ignored?

- All the **standard** notched fracture specimens have $\sigma_{xx} \approx 0$
- Generally considered: **line cracks**, as in
  
  - **linear elastic fracture mechanics**, LEFM (Griffith 1921)
  - **cohesive crack model**, CCM (Barenblatt 1959)

  - A line cut in $x$-direction in a field of homogeneous uniaxial stress $\sigma_{xx}$ causes **no stress change** in a continuum model

**What matters?** Fracture process, not the visible final crack

\[ \sigma_{xx} \approx 0 \]
Quasibrittle Materials

— brittle constituents, but inhomogeneity size and the RVE or FPZ are not $\ll$ structure size $D$.

Concrete (archetypical, 1970s) • tough ceramics
• fiber composites • rocks • bones • sea ice
• rigid foams • dental cements • dentine
• cartilage • wood • consolidated snow
• particle board • paper • carton • cast iron
• thin films • carbon nanotubes • cemented sand
• printed materials • fiber-reinforced concrete
• cold asphalt concrete • mortars • masonry
• stiff clay • silt • grouted soil • refractories
coal • oil and gas shales • various printed or
architected materials
• nacre • biological shells • and all
brittle materials on micro- and nano-scales.

At increasing size $D$, they all transition from ductile to brittle.

All: non-negligible material characteristic length.
Fracture Process Zone (FPZ) Size

10 km – Arctic Ocean ice cover as a 2D heterogenous medium consisting of thick floes of approx. 3 km size, embedded in thin ice matrix

5 m – Sea ice pushing on oil platform legs

0.5 m – Normal concrete

2 cm – Textile composites

2 mm – Gas or oil shale

(10^{-6} m – MEMS, polysilicone, embrittled metals—*We omit*)

– Quasibrittleness is a relative concept
Brittle Fracture Mechanics Founding in 1921 and Its Evolution into Ductile, Cohesive and Quasibrittle

A A Griffith 1921

Nonlinear fracture mechanics

Damaged mechanics

FPZ is long, wide, softening and tensorial

Brittle (dynamic)

Brittle or cohesive

FPZ = a point also a point large

ε or w
New finding: Effect of crack parallel stress $\sigma_{xx}$ on fracture energy $G_f$ of concrete is strong.

Data from regression of $3 \times 9 = 27$ experiments

3 data points – based on regression of $3 \times 9 = 27$ experiments

*M7 microplane prediction is calibrated only by $f_c$ and $G_{f,0}$*
I. Gap Test: New, yet simple and unambiguous, test of fracture at crack-parallel compression
Basic Idea of the Gap Test
– Achieve superposition in sequence
  (in statically determinate way)

1) \[ \sigma_{xx} \quad K_I = 0 \]

2) \[ K_I > 0 \]

\[ \sigma_{xx} \quad R + C \quad R + C \]

And keep \( \sigma_{xx} \) constant as \( K_I \) is applied
Novel Experiment: GAP TEST

Four advantageous key features:

1. At crack mouth, **plastic pads** to apply crack-parallel compression $C$.

2. A **gap** above end supports gets in contact only after the pads **yield**, to apply bending moment.

3. This way the test beam passes from one **statically determinate** system to another $\rightarrow$ simple and **unambiguous** evaluation.

4. The static determinacy and $C$ constancy enables the **size effect method** – a robust and unambiguous way to measure fracture energy $G_f$.

Typical Measured Load Evolution

Test Setup

The test setup consists of a Steel plate with a Plastic block (polypropylene) placed on it. The setup is designed to ensure that there is no lateral slip. DIC is used to verify the strain field for calibration.

The diagram shows the dimensions and the forces applied to the setup. The thickness of the Steel plate is labeled as 'b', and the gap size is labeled as 'a'. The nominal strain is measured using DIC, with a 0.39% rise and a 0.45% rise.

The graph on the left shows the relationship between the nominal strain and the stress on the pad (MPa). The x-axis represents the nominal strain, and the y-axis represents the stress in MPa.
Test Setup in More Detail

- Plastic pads (polypropylene)
- Extensometer
- Steel
- Gap
- Pads
- $F, \delta$
- $a$
Distinguish:

\[ \sigma_{\text{pad}} = \text{normal stress under the plastic pad, as measured} \]

\[ \sigma_{xx} = \text{inferred (by FE or DIC) crack-parallel stress at the notch tip FPZ, which is what matters for damage constitutive model.} \]

Typically: \[ \frac{\sigma_{xx}}{\sigma_{\text{pad}}} \approx 0.96 \]

Generally \[ (0.94, 0.97) \]
Gap Test for Tension

STAGE 1

\[ F = -2T \]

Steel glued

Pads yield in tension

Gap closed

STAGE 2

\[ F = 2R - 2T \]

Steel glued

Pads yield in tension

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Optimum Design of Polypropylene Pads

\[ H = \frac{\mu l^3 L}{4h^3} = \min \quad \text{(strip)} \]

\[ H = \frac{3\mu A^2}{8\pi h^3} = \min \quad \text{(circle)} \]

\( \mu, H = \) tangential stiffnesses of polypropylene and of pads
\( h, L, A = \) height, strip width and circle area of pads

Possible alternative to plastic pad: Tin (St)
II. Size effect method — essential part of gap test to measure material fracture energy $G_f$

Standard RILEM Recommendation, 1990. ACI-446 endorsed it and recommended it to ASTM C-09
For quasibrittle materials, distinguish the initial and total fracture energy, $G_f$ and $G_F$.

We consider only $G_f$.

$G_F$ is to be calculated from the ratio $G_F/G_f$ determined separately.
For Size Effect Test Method of $G_f$:
Specimens Geometrically Scaled (as $1 : 2 : 4$)

Depth:
$D = 4, 8, 16$ in.

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Required 1\textsuperscript{st} and 2\textsuperscript{nd} Order Small- and Large-Size Asymptotic Properties of Type 2 Size Effect on Fracture

For $D \rightarrow 0$:

$$\sigma_N \propto 1 - \frac{D}{D_s} - \ldots$$

For $D \rightarrow \infty$:

$$\sigma_N \propto \frac{1}{\sqrt{D}} \left( 1 - \frac{D_0}{2D} + \ldots \right)$$

Asymptotic Matching yields the


$$\sigma_N = \frac{\sigma_0}{\sqrt{1 + \frac{D}{D_0}}}$$

In detail: ZP Bažant (2004), PNAS, p. 13400
Size Effect Method to Measure Fracture Energy $G_f$ and Material Characteristic Length $c_f$

Transformation to linear regression:

$$Y = AX + C,$$

with

$$X = D, Y = \frac{E'}{g_0\sigma^2_N}$$

$$G_f = \frac{1}{A}, c_f = \frac{g_0}{g'_0} C$$

$$\sigma_N = B f'_t \left(1 + \frac{D}{D_0}\right)^{-1/2} = \sqrt{\frac{E'G_f}{g'(\alpha_0)c_f + g(\alpha_0)D}}$$

$g(\alpha) =$ dimensionless energy release function of LEFM

RILEM Standard Recommendation 1990
ACI-446 Recommendation

Size effect regression of 9 Gap Tests to get Gf for, e.g., the medium crack-parallel compression (the means and the scatter width)

\[ Y = AD + C \]

**Note the closeness of fit**

H. Nguyen, Z.P. Bažant, JAM, in press
Classical Work-of-Fracture Method of Measuring Total Fracture Energy $G_F$

Why was it not used?

1. J-integral varies with crack length x
2. Ambiguity of tail—ratios $G_F/G_f, f_t/f_1$ matter but vary
3. Tail hard to measure; stable postpeak required

Nakayama 1965
Hillerborg 1976
III.
Gap Test Results and Mesoscale Physical Mechanism
Found: Effect of crack parallel stress $\sigma_{xx}$ on fracture energy $G_f$ of concrete is strong.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fracture_energy_plot}
\caption{Effect of crack parallel stress $\sigma_{xx}$ on fracture energy $G_f$.}
\end{figure}

$G_f/G_{f,0}$ as a function of $\sigma_{xx}/\sigma_{xx,0}$ for different crack band models. M7 + crack band model shows a significant strengthening effect. LEFM, CCM, basic XFEM. Adjustment: $\frac{\sigma_{xx}}{\sigma_{pad}} \approx 0.96$.

3 data points – based on regression of $3 \times 9 = 27$ experiments.

M7 microplane prediction is calibrated by only $f_c$ and $G_{f,0}$. 

2 data points – based on regression of $2 \times 9 = 18$ experiments.
Material characteristic length $c_f$
dependence on crack-parallel stress $\sigma_{xx}$

Note: Approx. $c_f = 0.4$ FPZ length

Data from regression of $3 \times 9 = 27$ tests
Meso-Scale Mechanisms of $\sigma_{xx}$ Effect on $G_f$

Regimes of:

1) strengthening $G_f$
   
   $0 < \sigma_{xx} < 0.75\sigma_c$

2) weakening $G_f$

   $0.75\sigma_c < \sigma_{xx} < \sigma_c$

Static friction without slip enhanced by compression

Transverse widening of FPZ caused by inclined slips and splitting
IV. FE Extrapolations and Predictions for Fiber Concrete and Shale
The crack band model M7 with its default parameters was scaled to fit only:
1) uniaxial compression strength of the concrete, and
2) the measured basic fracture energy (at $\sigma_{xx} = 0$).

The predictions for the 2nd and 3rd data points were acceptable (10% error) and excellent (1% error).

Overall 7% error (which is normal for concrete)

So it makes sense to use microplane model M7 to make predictions for other situations and materials

Extrapolations Based on Microplane Model M7
Approximately Validated by Gap Tests
Effect of combined in-plane and out-of-plane stresses $\sigma_{xx}$ and $\sigma_{zz}$ on $G_f$ and $c_f$

Calculated for Gap Test Geometry

Fracture Energy

![Fracture Energy Graph]

Characteristic Length

![Characteristic Length Graph]

$\sigma_{xx} / \sigma_c$

$\sigma_{zz} / \sigma_c$

crack–parallel stress

Note the non-monotonic effect of $\sigma_{zz}$

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Comparisons with classical tensorial strength or damage models based on invariants

\[ \frac{G_f}{G_{f,0}} \]

CDPM2 concrete damage model (Grassl, Abaqus)

\[ \frac{G_f}{G_{f,0}} \]

Drucker-Prager \( I_1 \) and \( J_2 \) criterion (Abaqus)

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Fiber Reinforced Concrete (FRC) with 3% Dramix Steel Fibers vs. Plain Concrete

Fracture Energy

$G_f / G_{f0}$

$G_{f0} = 86.7 \text{ N/m}$

Plain concrete

$G_{f0} = 143.1 \text{ N/m}$

FRC

Crack–parallel compression $\sigma_{xx} / \sigma_{xx,c}$

Characteristic Length

$C_f / C_{f0}$

$C_{f0} = 11.2 \text{ mm}$

Plain concrete

$C_{f0}$

FRC

Calculated with M7 for concrete and M7f for FRC

– tests yet to be made!

Gap Test for Gas Shale Predicted with Spherocylindrical Anisotropic Microplane Model

Fracture Energy

Characteristic Length

Crack–parallel compression $\sigma_{xx} / \sigma_{xx,c}$

**NOTE:** After the Virus, collaborative tests will resume with LANL (Luke Fray, Hari Viswanathan, Bill Carey and Esteban Rougier)

*Li, Cunbao,...Bažant, JMPS 103, 155-178 (2017)

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press
Strong effect of history (or path) of loading, calculated for concrete

\[ \sigma_N \propto K_I \]

First \( \sigma_N^* \)
Then \( \sigma_{xx} \)

Reversed:
First \( 0.62 \sigma_N^* \)

\[ 1.81 \sigma_N^* \]

Hence, no universal formula for \( G_f \) as function of \( \sigma_{xx} \) exists!

H. Nguyen, .... Z.P. Bažant, PNAS, in press; JAM July 2020
V. Alternative Tests Considered and Retrospective
Alternative setup – proportional loading

Problem: \( \sigma_{xx} \) is not constant

Evaluating \( G_f \) would require optimal FE fitting with crack-band damage model
Tschegg’s 1995 wedge-splitting test with crack-parallel compression

— pioneering idea!*
Tests showed some effect but were ambiguous because:
- work-of-fracture method was used
- no size effect was tested
- non-uniform stress field
- bending and friction from weight of hydraulic jacks

*Ignored by fracture community due to lack of simplicity and high scatter
Retrospective

• Elasto-plastic metals: Stress triaxiality in an annulus around crack tip, led to extra parameter $Q$ accounting for compressive T-stress ($= \sigma_{xx}$), and an increase of $J$-integral based on HRR field for ductile fracture (Shih, Hancock, Tvergaard,... 1990s).

• Crack path deflection due to T-stress in LEFM was solved by Cotterell and Rice (1980)

**But both problems are different.**
VI.
New Perspective of Fracture Mechanics of Quasibrittle Materials
Can the existing FE programs for LEFM, CCM, XFEM and phase-field model be used? Hardly, because:

— **Path dependence of damage** is even stronger than it is in plasticity

— **Different** formulae would be needed for **different** materials, stress ratios $\sigma_{zz}/\sigma_{xx}$, plane strain, ...
• To adjust $G_f$ of the cohesive crack model is ambiguous since many different combinations of tensile strength $f_t$ and softening slope are possible, and adjusting ratio $G_F/G_f$ is unclear.

• Expected: A strong $\sigma_{xx}$ effect in anisotropic fiber-polymer composites – e.g., a pressurized fuselage is under high biaxial tension.

• Delamination fracture is sure to depend on $\sigma_{xx}, \sigma_{zz}$. 
Paramount Problem: **Realistic Tensorial Damage Model**

for concrete, fit 22 different benchmark triaxial tests (microplane model does)*

- uni-, bi-, tri-axial
- splitting strength
- proportional or nonproportional
- post-peak softening at FPZ scale

- **Plus Vertex Effect:**
  - Tangential stiffness at start of torsion vs. axial strain of compressed cylinder rotating principal stress axes

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**Tensorial invariant-based models**

Experiment: Caner-Bažant 2002

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**Initial Compressive Strain**

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**Non-associated** dilatancy of frictional slip (captured by M7)
- tension-compression-torsion
- unloading-reloading cycles...

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* FC Caner, ZP Bažant (2013), JEM, p. 1714

** FC Caner, ZP Bažant, J Cervenka, J. (2002). JEM, 24-33; ZP Bažant (!982), J Eng Mech ASCE 109, 849--865
A hundred different damage constitutive models for concrete exist!

- All can work for one crack:
- All can fit Mode I test of $F(u)$ softening curve, using:
  - scalar softening $\sigma_{yy} = f(\kappa)$
  - $I_1^e$ volumetric softening
  - $J_2^e$ deviatoric softening (~von Mises)
  - Mazars’ $\|\varepsilon^+\|$ one-param. softening, etc.

But should also fit:

- basic material test data and benchmark triaxial tests (with approx. FPZ size)
- multiple interacting cracks
- gap tests

parallel cracks
Both the tensorial nature of the FPZ and the minimum possible spacing of parallel cracks can be effectively captured by the crack-band model.

Eigenstrain $\varepsilon^0$

\[ \sigma_{xx} \]

Localizing

Not

Devil's Postpile, Sierra
Effects of $\sigma_{xx}, \sigma_{zz}$ in fracking of shale – probably great

3 km deep, vertical crack faces are under high tectonic and overburden crack-parallel stresses $\sigma_{xx}, \sigma_{zz}$, combined by body forces applied by diffusion pressure gradients.

Simulated vertical hydraulic cracks in shale with preexisting weak layers if $G_f$ is constant — no localization!

(Rahimi,...Bažant, PNAS 2019, p. 1532)
Continue using Mohr failure envelope for large crack-parallel stress $\sigma_{xx}$? — in doubt
—widely used in geophysics and in fracking studies.

Envelopes from gap tests, extended by FE

$$\sigma_I = \text{transverse tensile stress in FPZ, via FE}$$
$$\sigma_{III} = \sigma_{xx} = \text{crack-parallel compressive stress at FPZ}$$
Gap Test proves that shear fracture of beams and slabs must be analyzed taking into account a large tensorial FPZ.

Despite 40 years of studies, CCM, LEFM never succeeded; $K_I \to 0$ at $V_{\text{max}}$. This was one experience that motivated the gap test.

Promise of Multiscale Approach

• Tensorial damage band shrunken into a line (Remmers, de Borst, Needleman, IJF 2013). Yields a cohesive crack with an embedded subscale tensorial damage band.

• But tensorial damage law remains a big question.

• Calibration by data fits not yet demonstrated.

• Conceptual problem: Minimum crack spacing will not be enforced automatically. Parallel cracks?
Promise of Phase-Field Model

– computationally enticing, alternative. But:

• **Single** parameter damage—ineffective.

• Needs a realistic tensorial model for softening damage, such as microplane, *validated* and calibrated by existing triaxial material tests of diverse types, esp. of FPZ size

• **Transition from distributed damage** to isolated bands? Minimum spacing of parallel cracks?
In 1980s, forces $F$ measured on legs of oil platforms in Prudhoe Bay were an order-of-magnitude less than FE predictions with buckling.

— **One (incomplete) explanation:** Size effect  

— **Now 2nd explanation:** Crack-parallel compression in large FPZ reduced $G_f$ (or $K_{Ic}$) near 0.

In glaciers — $\sigma_{xx}$ in vertical cracks, depth >1 km

Expected: Effect of $\sigma_{xx}$, $\sigma_{zz}$ on fracture of bone and biomaterials

Why? Observed size effect implies large FPZ

*Hip fracture, tooth fracture, ... often occurs at high $\sigma_{xx}$*

From size effect:
Cohesive softening law of bone has a long tail

Bone tests by Bažant, Kim, Yu (IJF 2013)
Tests of PEEK at Northwestern (Bažant-Kim,..., IJF 1999)

- Kink-bands in fiber composites are a fracture propagation problem with strong size effect

- $\sigma_{xx}$ effect — unknown but likely.

Kink band micro-buckling with shear fractures leads to size effect
Large FPZ suggests:

**effect of crack-parallel** $\sigma_{xx}, \sigma_{zz}$ **on subcritical crack growth**

1) **Charles-Evans law** for static fatigue crack growth in rocks or ceramics, and

1) **Paris law** for cyclic fatigue crack growth in quasibrittle materials.

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Bažant & Xu tests (ACI J 1991), for 3 specimen sizes

Size effect proves large FPZ

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Tensorial Aspect of Shear Fracture

In concrete, compression changes a planar shear crack to an inclined conical shear crack.

Inexplicable if constant $G_f$ or $K_c$ govern.

Torsional shear tests
Bažant, Prat, Tabbara, ACI J. 1990, 12-19
• Seismically sliding **geological faults** are very thin (< 1 mm), but if the FPZ at front is not, then $\sigma_{xx}, \sigma_{zz}$ should matter for their propagation.

• Cracks growing in **prestressed** concrete have a different $G_f$.

• The $G_f$ variation is generally not expected in **metals**, fine-grained ceramics or polysilicon, since their FPZ is of micrometer scale. But it could matter for **MEMS**, or devices < $10^{-4}$ m.
Conclusions

• **Gap Test – simple, unambiguous** interpretation – one *statically determinate* system transits to another – size effect test of $G_f$ possible

• **FPZ – Tensorial !**
  Effective simple approach – FE crack band model.

• The **CCM and LEFM (and XFEM)** are approximations of rather limited applicability.

• **LEFM and CCM** remain *essential* for understanding and teaching fracture mechanics, and for providing accurate *benchmark solutions* of special cases which tensorial damage mechanics must match.

• The evidence from the gap test looks compelling but so far is *scant* and deserves broad scrutiny. *Extensive testing and calibration* will be needed – for most quasibrittle materials.
Afterthought

Many hot research subjects become closed in a few decades. But, like turbulence, fracture mechanics is different. This formidable subject has been researched for a century, and probably will for another century.

Thanks for listening!