CIV_ENV 495-35
Plasticity and Limit Analysis
Fall Quarter 2018

Instructor	Prof. James P. Hambleton Office: Tech A122 Office hours: By appointment Phone: (847)491-4858 Email: jphambleton@northwestern.edu
Class Times*	Tuesday and Thursday, 11:00-12:20pm
Location	Frances Searle Building 3220
Class Website	Northwestern Course Management System (Canvas) http://www.it.northwestern.edu/education/login.html
Required Textbook	None
Suggested Reading	Chen, W.-F. (1975). *Limit Analysis and Soil Plasticity*. Elsevier, Amsterdam. Chen, W.-F., and Han, D.J. (1978). *Plasticity for Structural Engineers*. Springer-Verlag, New York. Calladine, C.R. (1985). *Plasticity for Engineers*. Ellis Horwood Limited, Chichester, UK. Davis, R. O., and Selvadurai, A. P. S. (2005). *Plasticity and Geomechanics*. Cambridge University Press, Cambridge.
Prerequisites	Familiarity with elementary mechanics, including the concepts of stress, strain, and equilibrium Familiarity with linear algebra and basic programming

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Course Objectives

- Introduce fundamental theory of *plasticity*, including the concepts of yielding and plastic flow in materials and, by extension, the concepts of limit (collapse) loads and collapse mechanisms in boundary value problems
- Introduce the various *techniques available for computing limit loads*, including the slip-line method (method of characteristics), limit equilibrium, analytical and numerical limit analysis, and the finite element method†
- Understand and apply *limit analysis* as a method for evaluating rigorous bounds on limit loads for stability problems in engineering
- Program basic finite element limit analysis (FELA) codes
- Introduce advanced concepts

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* Subject to change due to scheduling conflicts; updated class times will be announced in class and on Canvas.

† The course Civ_Env 495-19 Computational Geotechnics addresses the implementation of plasticity, and aspects of material nonlinearity, in the displacement-based finite element method.

Course Outline

Week 1	
Class 1	Stress and equilibrium
Class 2	Strain and strain rates
Week 2	
Class 3	Mechanical properties of soils and solids; perfect plasticity
Class 4	Yield conditions and flow rules
Week 3	
Class 5	Slip-line method
Class 6	Limit theorems
Week 4	
Class 7	Lower bound limit analysis: concepts
Class 8	Lower bound limit analysis: analytical solutions
Week 5	
Class 9	Lower bound finite element limit analysis (FELA)
Class 10	Mathematical optimization: linear programming
Week 6	
Class 11	Programming lower bound FELA
Class 12	Upper bound limit analysis: concepts
Week 7	
Class 13	Upper bound limit analysis: analytical solutions
Class 14	Programming upper bound FELA
Week 8	
Class 15	Limit analysis versus limit equilibrium
Class 16	Selected applications: bearing capacity, slope stability, …
Week 9	
Class 17	Selected applications
Class 18	Possibilities and limitations of limit analysis; non-associated flow
Week 10	
Class 19	Advanced topics: generalized forces, steady-state flow problems, …
Class 20	Advanced topics

Course Assessment

Grades are determined based on the following components, weighted as indicated:

- 10% Class participation
- 15% Project #1: Reviewing a contemporary work on plasticity and limit analysis
- 15% Project #2: Programming a lower bound finite element limit analysis code
- 15% Project #3: Solving an upper-bound problem using multi-variable optimization
- 45% Final Project