ESAM SEMINAR SPEAKER

Is the Ott-Antonsen Manifold Attracting?

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March 2, 2020 • 4 pm
M416 (ESAM Conference Room), Tech

The Kuramoto model is a paradigm for studying oscillator networks with interplay between coupling tending towards synchronization, and heterogeneity in the oscillator population driving away from synchrony. In continuum versions of this model, an oscillator population is represented by a probability density on the circle. In 2007, Ott and Antonsen identified a special class of densities which is invariant under the dynamics, and for which the dynamics are low-dimensional and analytically tractable. The reduction to this so-called OA manifold has been used to analyze the dynamics of many variants of the Kuramoto model. We will address the fundamental question of whether the OA manifold is attracting in the full state space. We show that for models with a finite number of populations, the OA manifold is not attracting in any sense; moreover, the dynamical behavior off the OA manifold is typically more complicated than on the OA manifold. The OA manifold consists of Poisson densities; a simple extension of the OA manifold consists of averages of pairs of Poisson densities. We prove that the hyperbolic distance in the disc between the centroids of each Poisson pair is a dynamical invariant. These conserved quantities, defined on the double Poisson manifold, give a way to measure the distance to the OA manifold, and this metric is dynamically invariant. This invariance implies that the OA manifold is not attracting. As an application, we show that chimera states, in which some but not all oscillator populations are in sync, can never be stable in the full state space, even if stable within the OA manifold. More importantly, perturbations of chimera states will have more complex dynamics off the OA manifold; for example, perturbing from a chimera state which is stable in the OA will yield limit cycle dynamics, and perturbing from a periodic chimera state stable in the OA ("breathing chimera") will yield quasi-periodic dynamics. More generally, this framework allows for the analysis of multi-population continuum Kuramoto networks beyond the OA manifold, with the potential to reveal more intricate dynamical behavior than has previously been observed for these networks.

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