NEW PERSPECTIVE OF FRACTURE MECHANICS INSPIRED BY NOVEL TEST WITH CRACK-PARALLEL COMPRESSION

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NU-TAM WEBINARS ON BREAKTHROUGHS IN MECHANICS

NORTHWESTERN UNIVERSITY, JUNE 2, 2020

BASED ON: PNAS—IN PRESS (JUNE). EXTENSION: JAM (JULY 2020)
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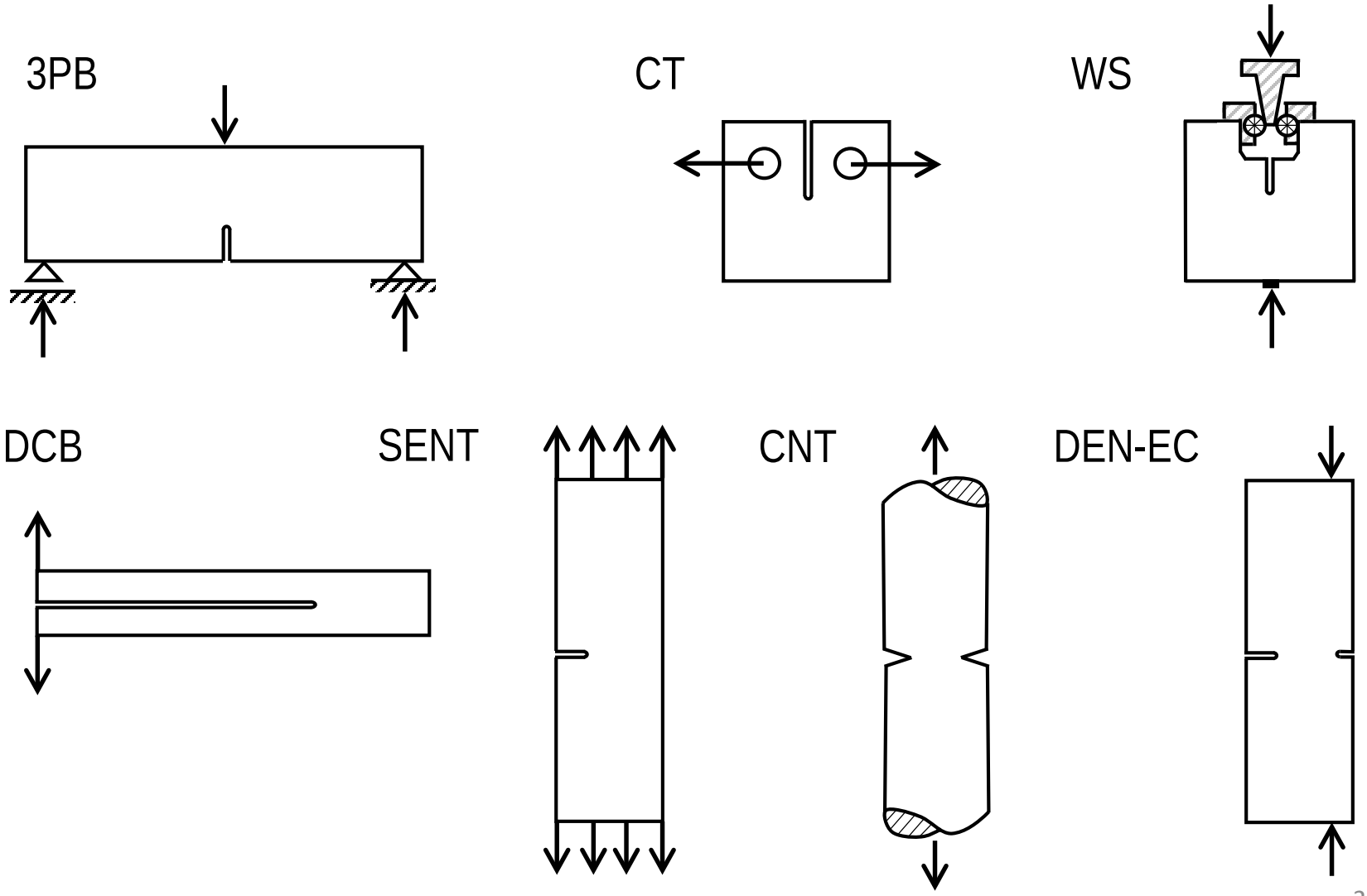
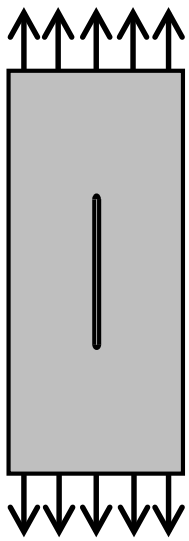
Standard Fracture Test Specimens
— negligible crack-parallel normal stress



Why has the crack-parallel stress σ_{xx} been ignored?

- All the standard notched fracture specimens have $\sigma_{xx} \approx 0$
- Generally considered: line cracks, as in
  - linear elastic fracture mechanics, LEFM (Griffith 1921)
  - cohesive crack model, CCM (Barenblatt 1959)
- A line cut in x-direction in a field of homogeneous uniaxial stress σ_{xx} causes no stress change in a continuum model

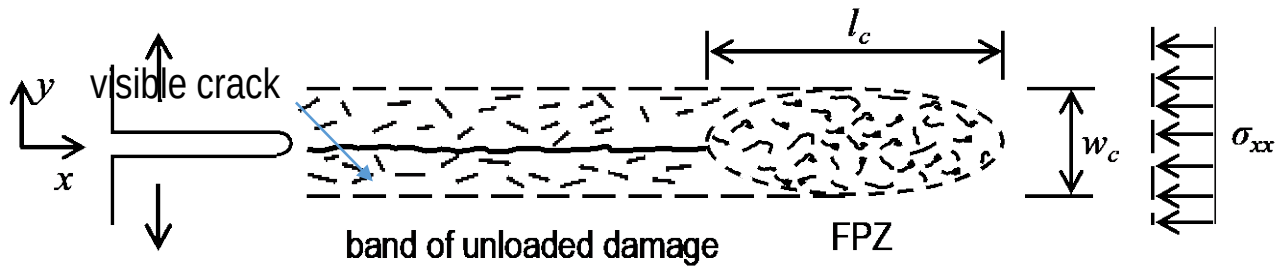
Why:



σ_{xx}

What matters?

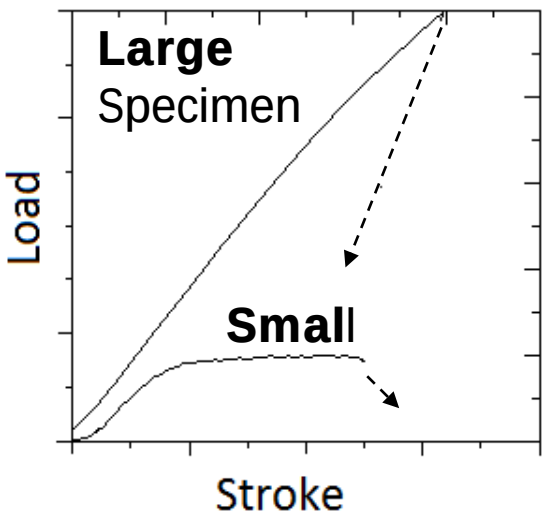
Fracture process, not the visible final crack



Quasibrittle Materials

— brittle constituents, but inhomogenity size and the RVE or FPZ are not << structure size D.

Concrete (archetypical, 1970s) ▪ tough ceramics ▪ fiber composites ▪ rocks ▪ bones ▪ sea ice ▪ rigid foams ▪ dental cements ▪ dentine ▪ cartilage ▪ wood ▪ consolidated snow ▪ particle board ▪ paper ▪ carton ▪ cast iron ▪ thin films ▪ carbon nanotubes ▪ cemented sand ▪ printed materials ▪ fiber-reinforced concrete ▪ cold asphalt concrete ▪ mortars ▪ masonry ▪ stiff clay ▪ silt ▪ grouted soil ▪ refractories coal ▪ oil and gas shales ▪ various printed or architectured materials ▪ nacre ▪ biological shells ▪ and all brittle materials on micro- and nano-scales.



At increasing size D, they all transition from ductile to brittle.

All: non-negligible material characteristic length.

Fracture Process Zone (FPZ) Size

10 km – Arctic Ocean ice cover as a 2D heterogenous medium consisting of thick floes of approx. 3 km size, embedded in thin ice matrix

5 m – Sea ice pushing on oil platform legs

0.5 m – **Normal concrete**

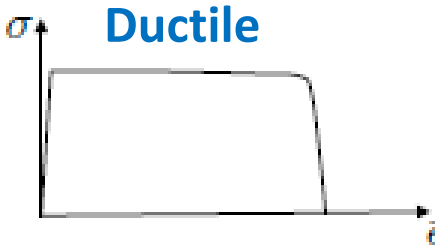
2 cm – Textile composites

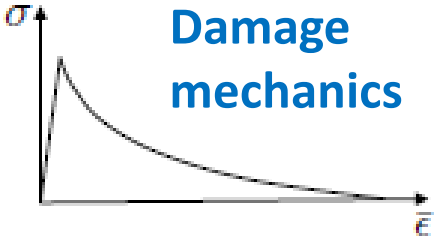
2 mm – Gas or oil shale

(10^{-6}m – MEMS, polysilicone, embrittled metals—*We omit*)

– Quasibrittleness is a <u>relative concept</u>

Brittle Fracture Mechanics Founding in 1921 and Its Evolution into Ductile, Cohesive and Quasibrittle

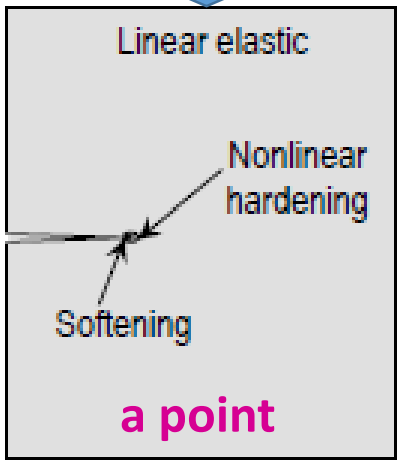


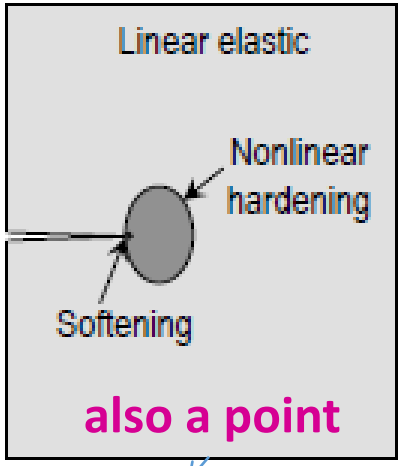


Nonlinear fracture mechanics

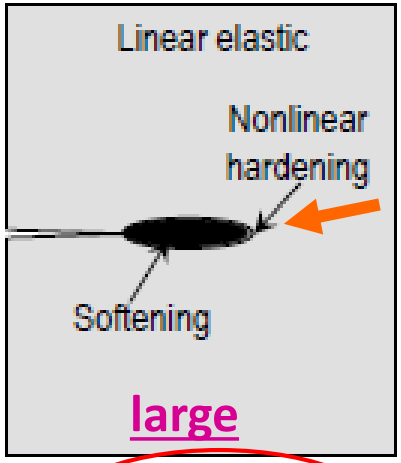
1950 – 1980

1980 –





or



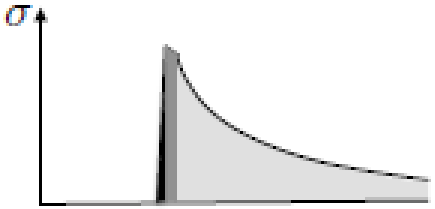
FPZ is long, wide, softening and *tensorial*

FPZ = a point

also a point

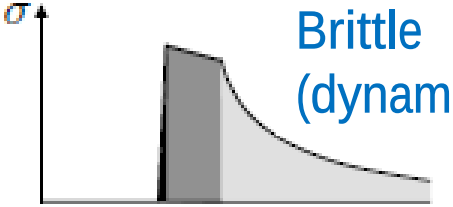
large

Brittle

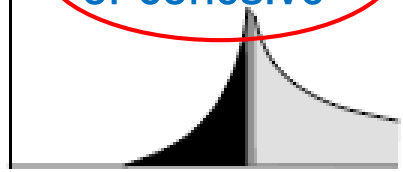


Ductile

Brittle (dynamic)

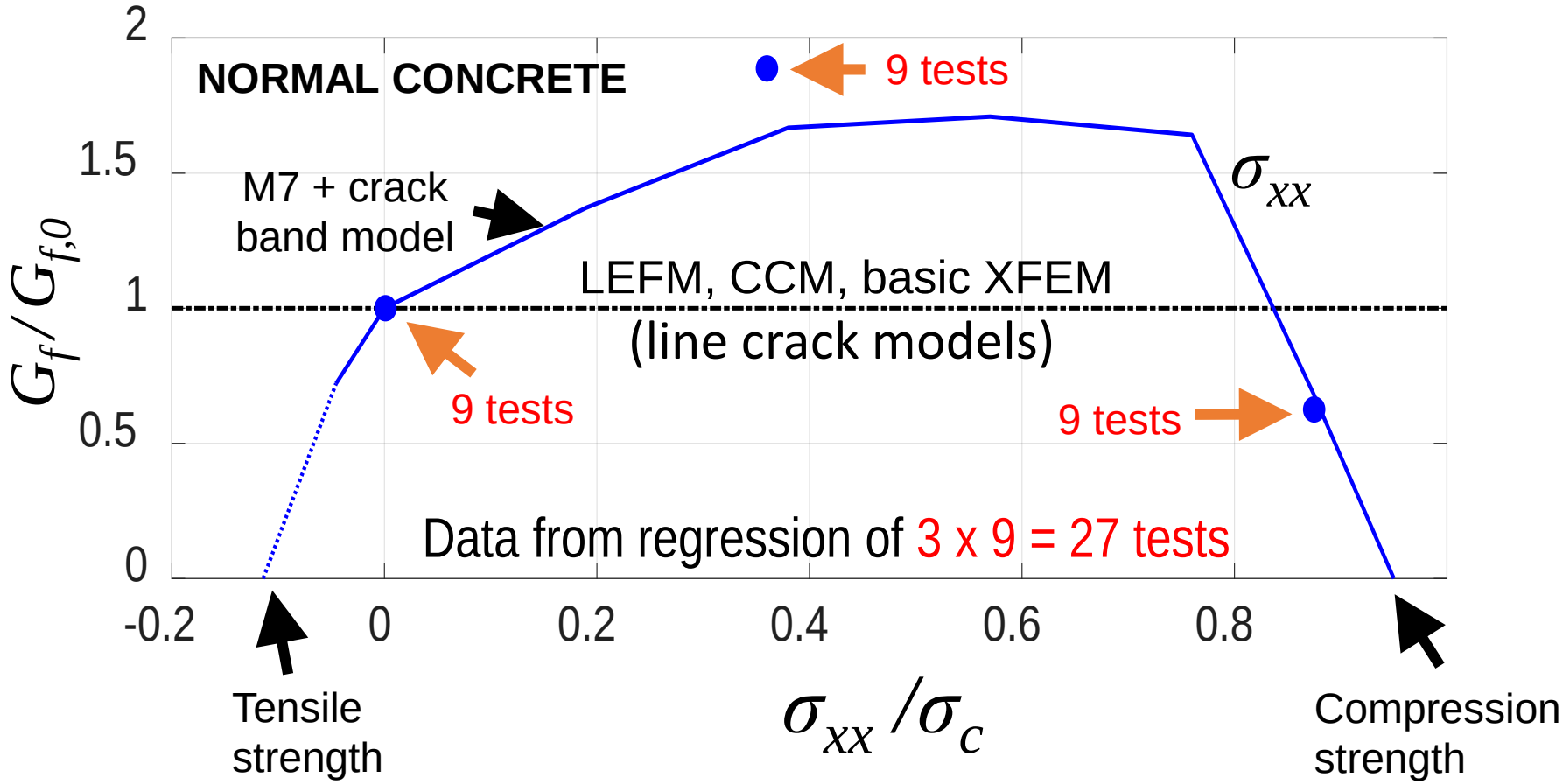


Quasibrittle or cohesive



ε or w

New finding: Effect of crack parallel stress σ_{xx} on fracture energy G_f of concrete is strong



3 data points – based on regression of 3 x 9 = 27 experiments

M7 microplane prediction is calibrated only by f_c and $G_{f,0}$

I.

Gap Test:

New, yet

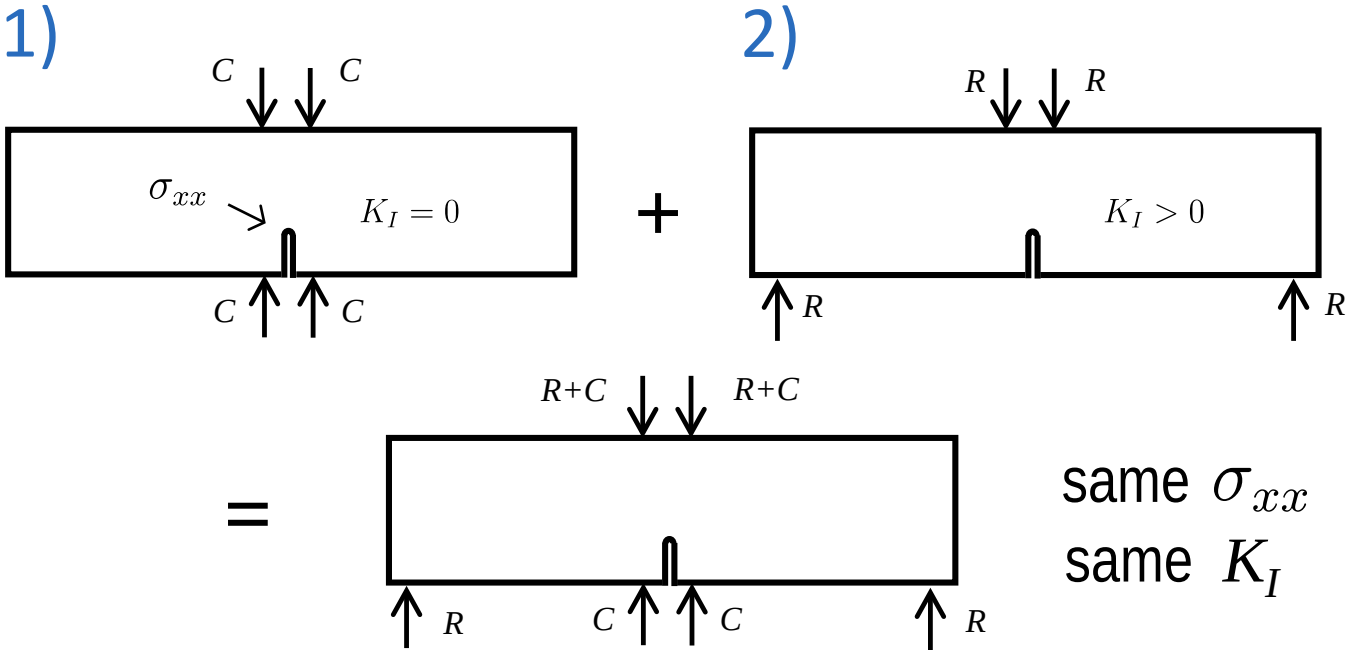
simple and unambiguous,

test of fracture at

crack-parallel compression

Basic Idea of the Gap Test

– Achieve superposition in sequence (in statically determinate way)



same σ_{xx}
same K_I

And keep σ_{xx} constant as K_I is applied

Novel Experiment: GAP TEST

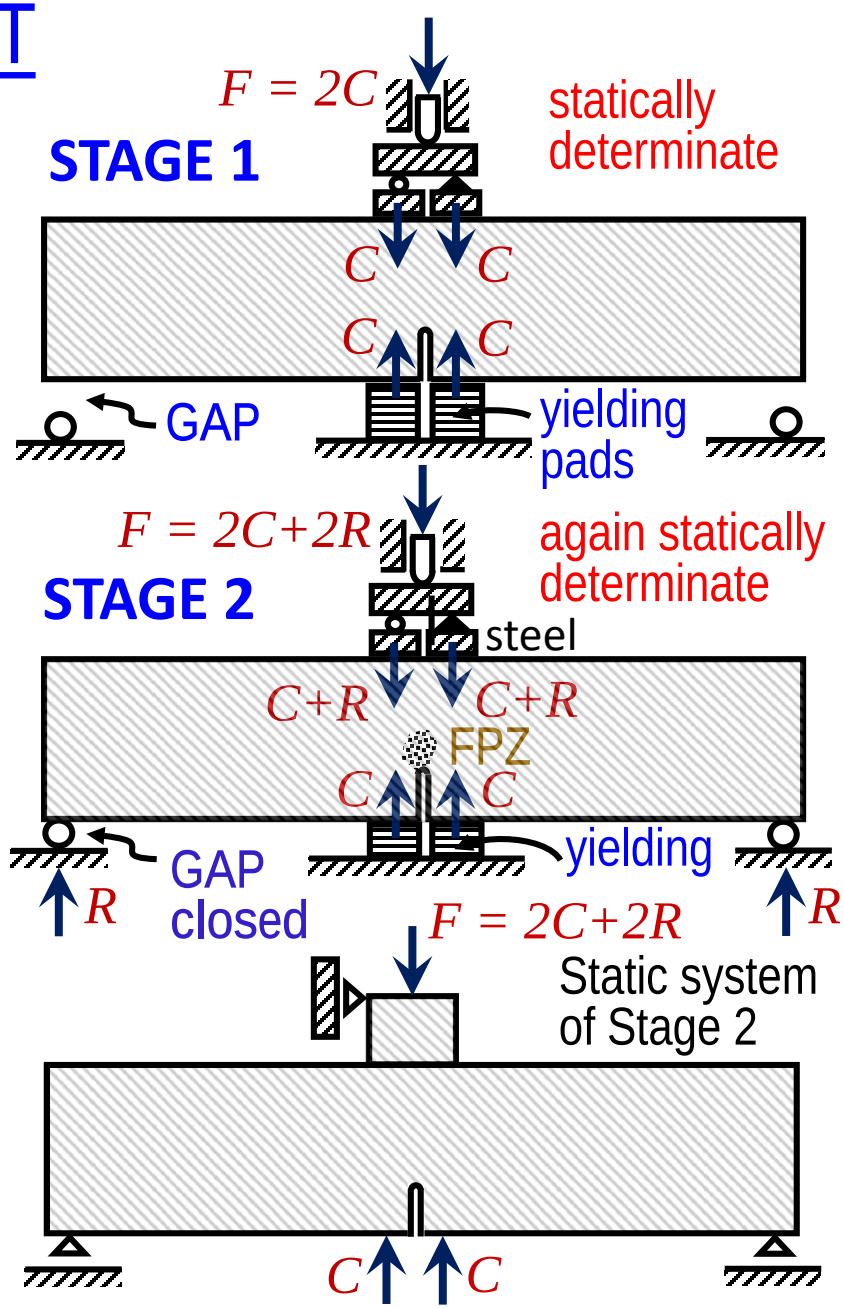
Four advantageous key features:

1. At crack mouth, **plastic pads** to apply crack-parallel compression C.

2. A gap above end supports gets in **contact only after the pads yield**, to apply bending moment.

3. This way the test beam passes from one statically determinate system to another → simple and **unambiguous** evaluation.

4. The static determinacy and C constancy enables the **size effect method** – a robust and unambiguous way to measure fracture energy G_f



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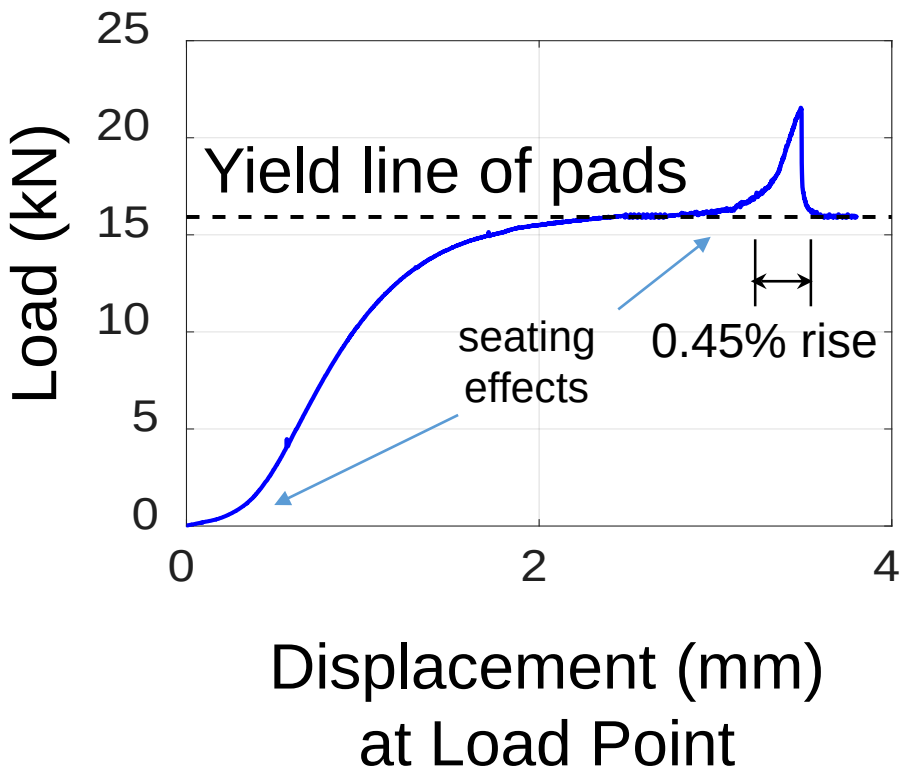
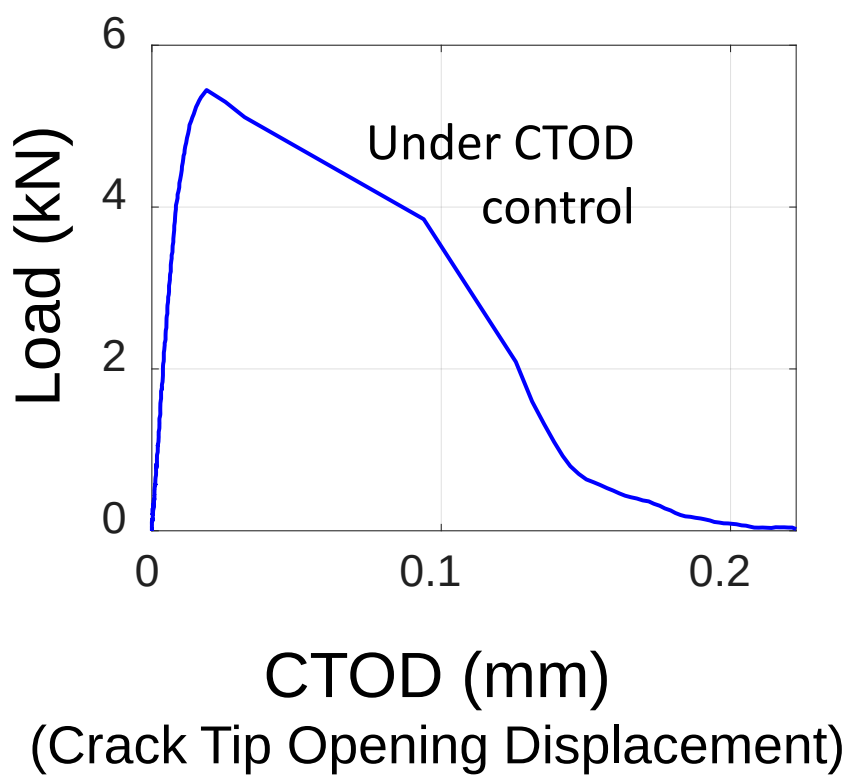
Typical Measured Load Evolution





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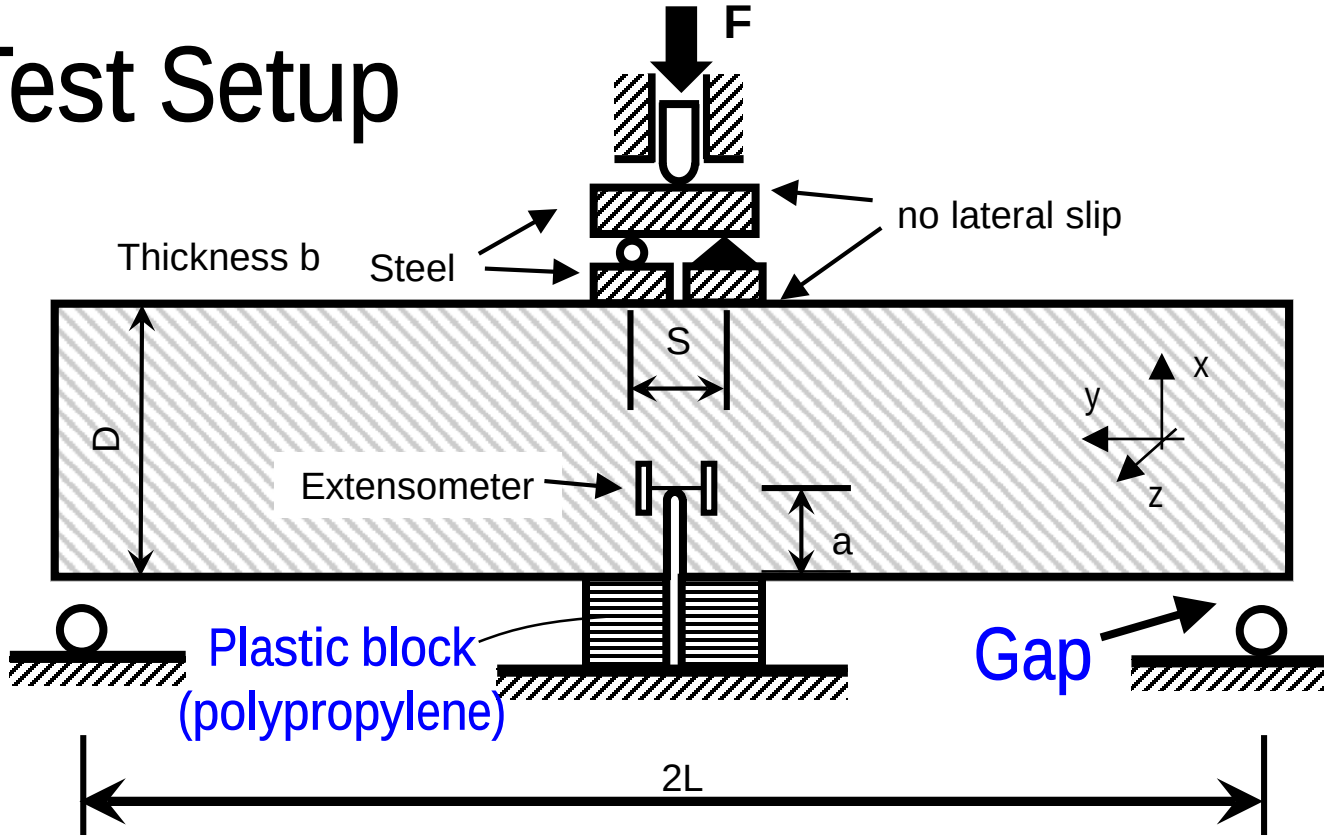
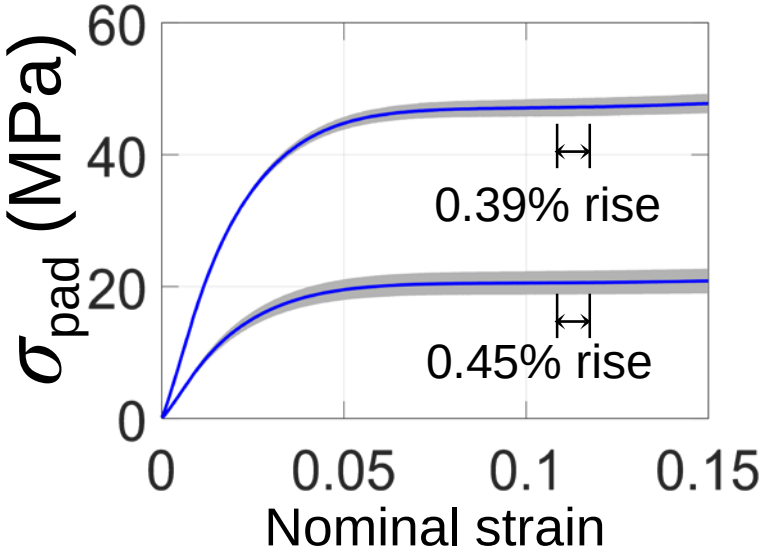
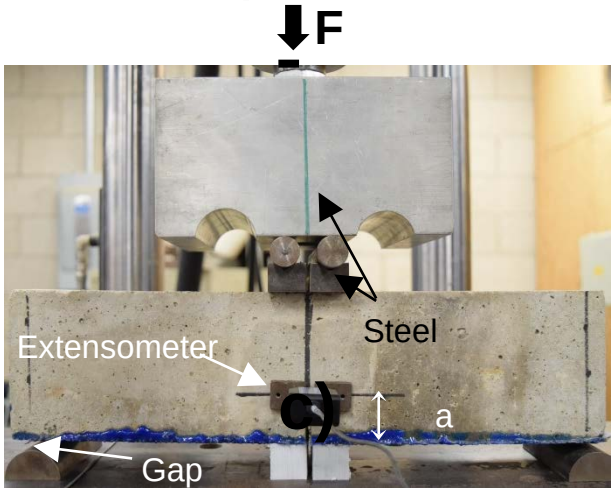
Test Setup



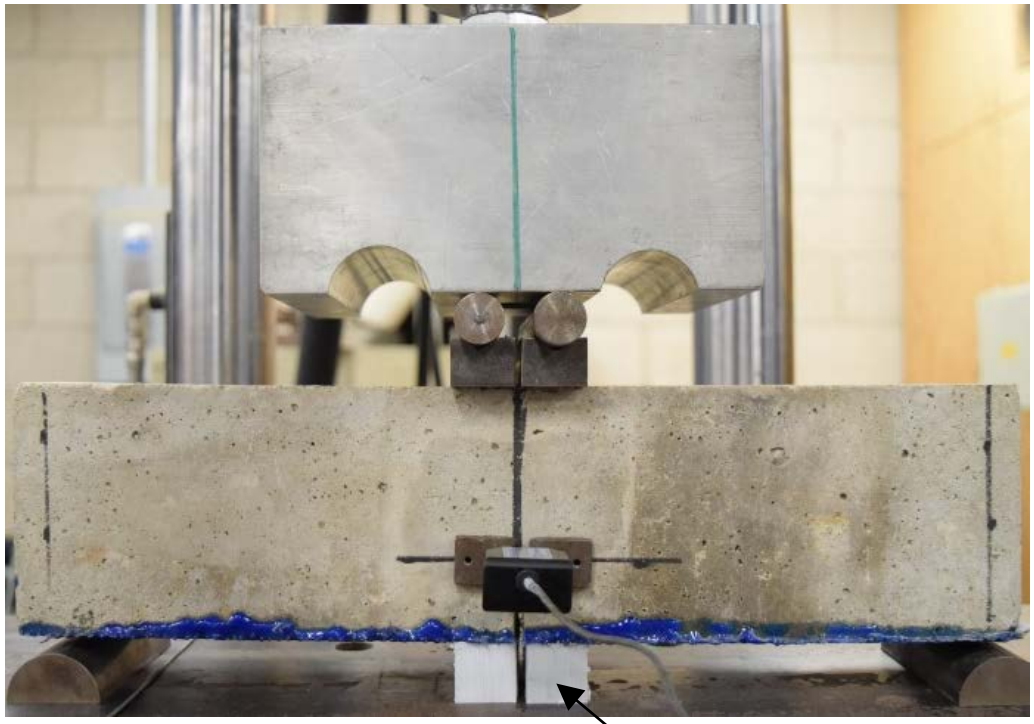
DIC used to verify strain field for calibration



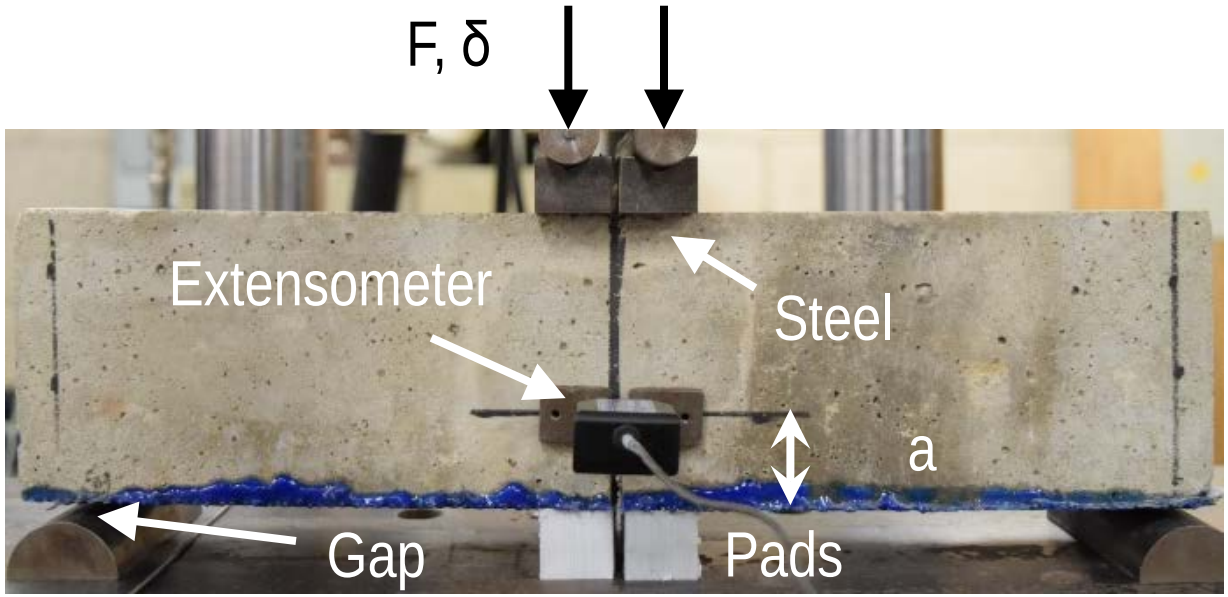


Plastic blocks (polypropylene)

Test Setup in More Detail



Plastic pads (polypropylene)



Computed Field of Principal Stress Vectors

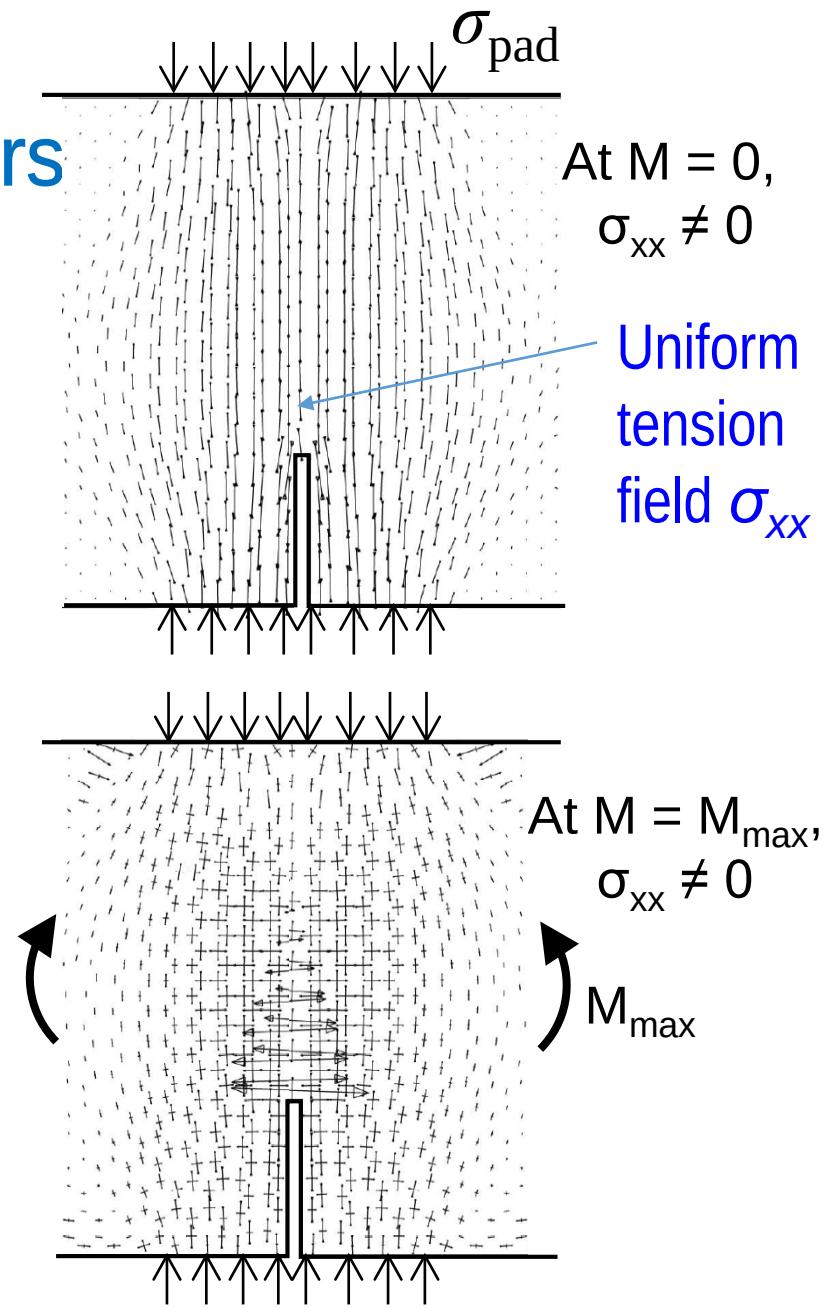
Distinguish:

σ_{pad} = normal stress under the <u>plastic pad</u>, as measured

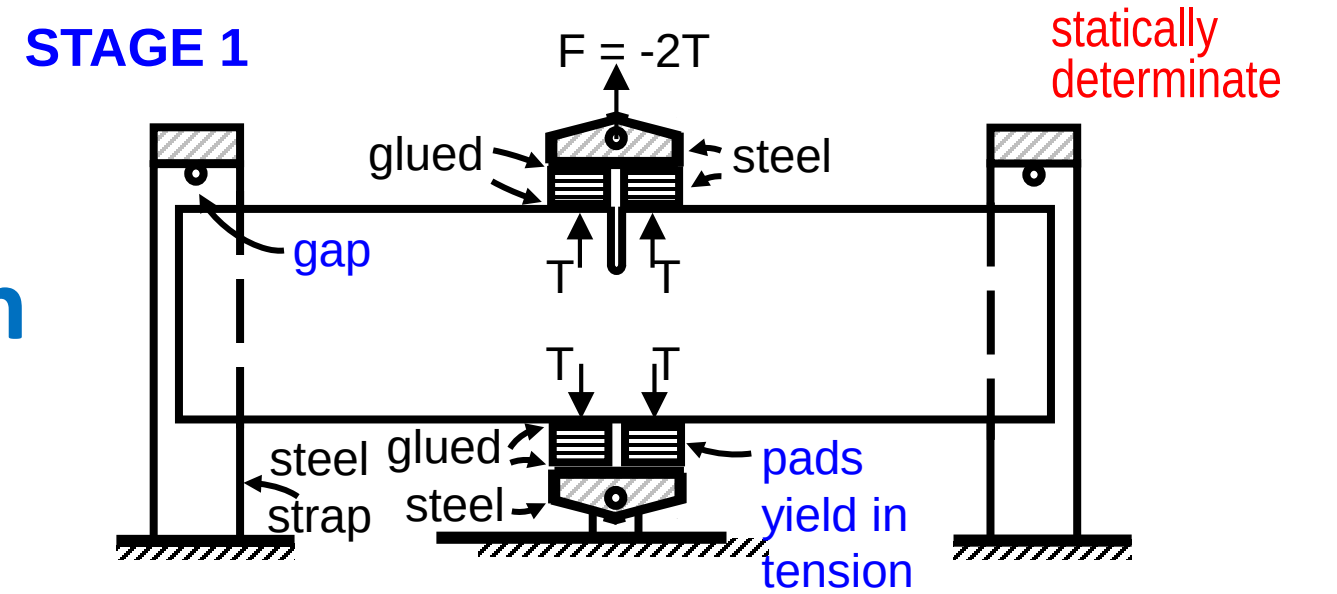
σ_{xx} = inferred (by FE or DIC) <u>crack-parallel stress</u> at the notch tip FPZ, which is what matters for damage constitutive model.

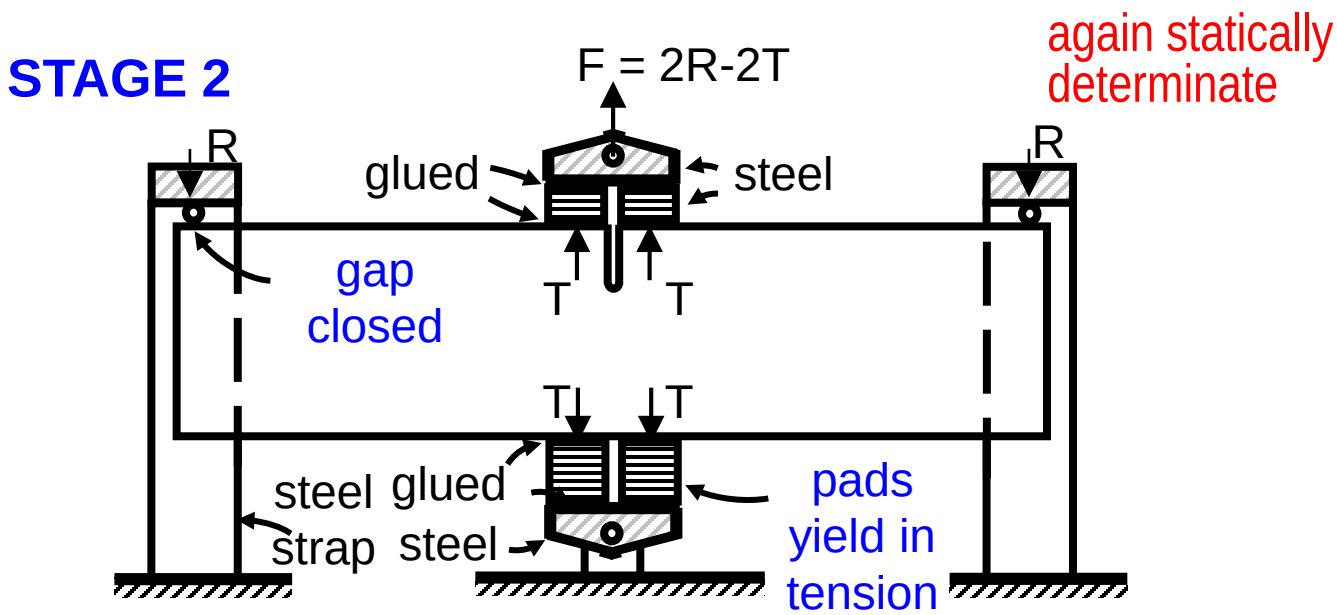
$$\text{Typically:} \quad \frac{\sigma_{xx}}{\sigma_{pad}} \approx 0.96$$

$$\text{generally} \quad \in (0.94, 0.97)$$



Gap Test for Tension





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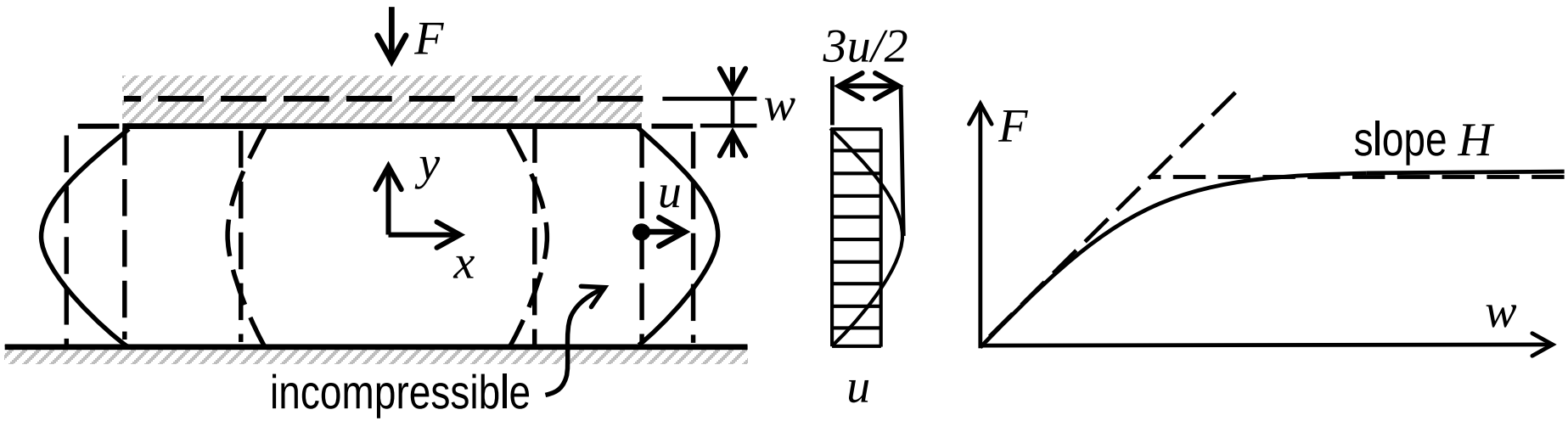
Optimum Design of Polypropylene Pads



$$H = \frac{\mu}{4}\,\frac{l^3 L}{h^3} = \min \quad \text{(strip)}$$

$$H = \frac{3\mu}{8\pi}\,\frac{A^2}{h^3} = \min \quad \text{(circle)}$$

$\mu, H =$ tangential stiffnesses of polypropylene and of pads

$h, L, A =$ height, strip width and cirle area of pads

Possible alternative to plastic pad: Tin (St)

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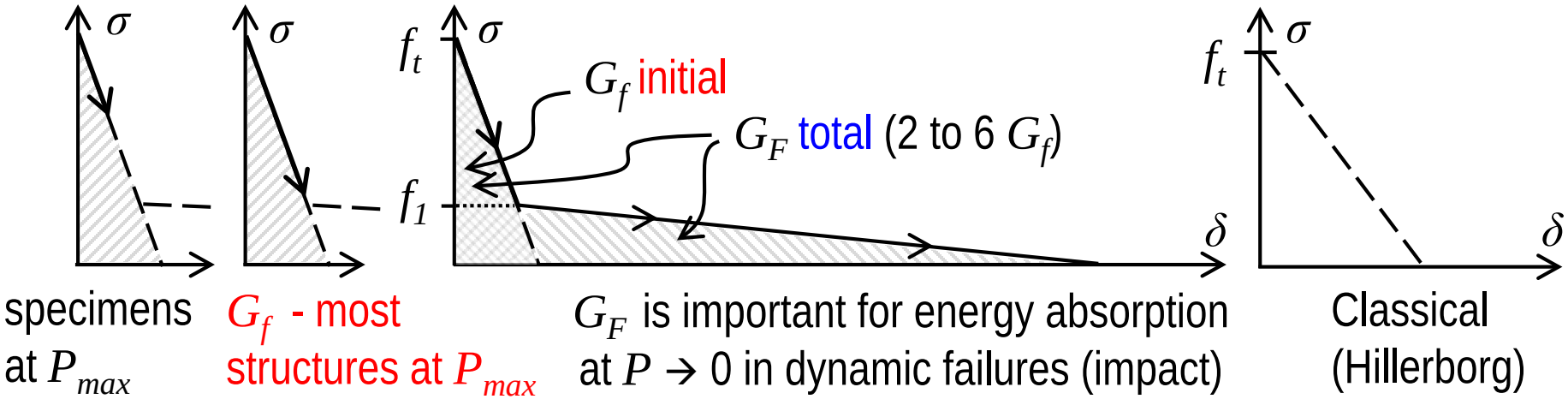
II.
Size effect method — essential part of gap test to measure material fracture energy G_f

Standard RILEM Recommendation, 1990.
ACI-446 endorsed it and recommended it to ASTM C-09

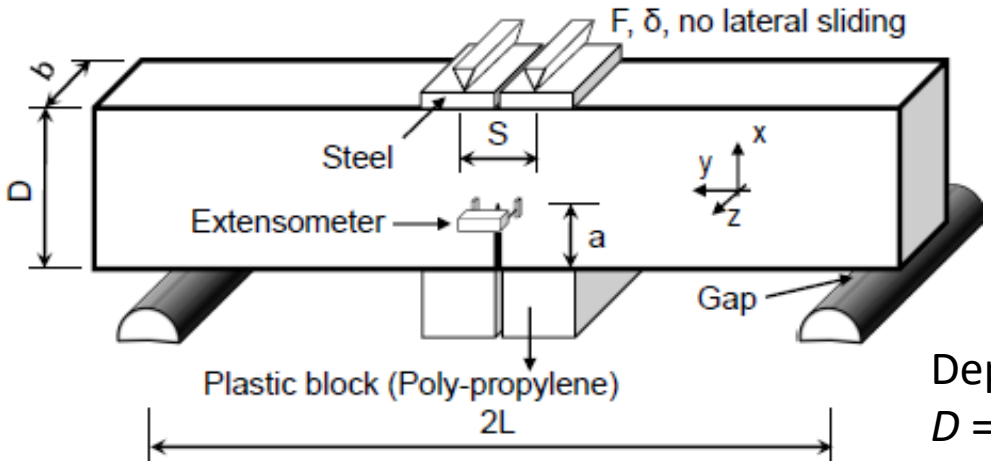
For quasibrittle materials, distinguish the initial and total fracture energy, G_f and G_F



We consider only G_f.

G_F is to be calculated from the ratio G_F/G_f determined separately.

For Size Effect Test Method of G_f: Specimens Geometrically Scaled (as 1 : 2 : 4)



Depths:
D = 4, 8, 16 in.

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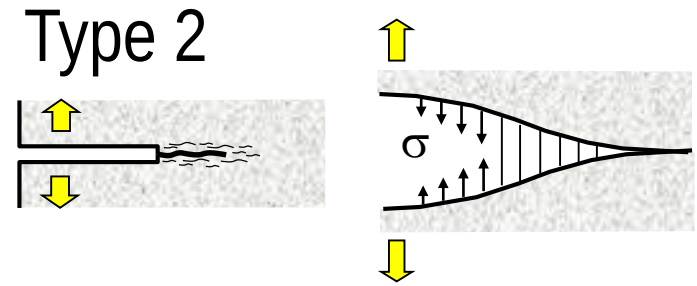
Required 1st and 2nd Order Small- and Large-Size Asymptotic Properties of Type 2 Size Effect on Fracture

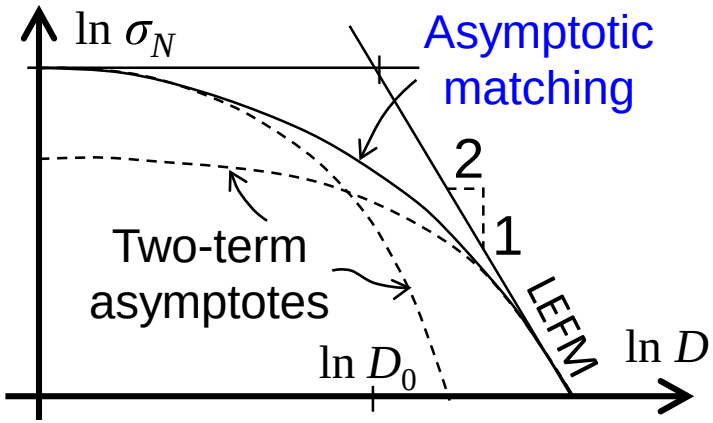
For $D \to 0$:

$$\sigma_N \propto 1 - \frac{D}{D_s} - \ldots$$

For $D \to \infty$:

$$\sigma_N \propto \frac{1}{\sqrt{D}}\left(1 - \frac{D_0}{2D} + \ldots\right)$$





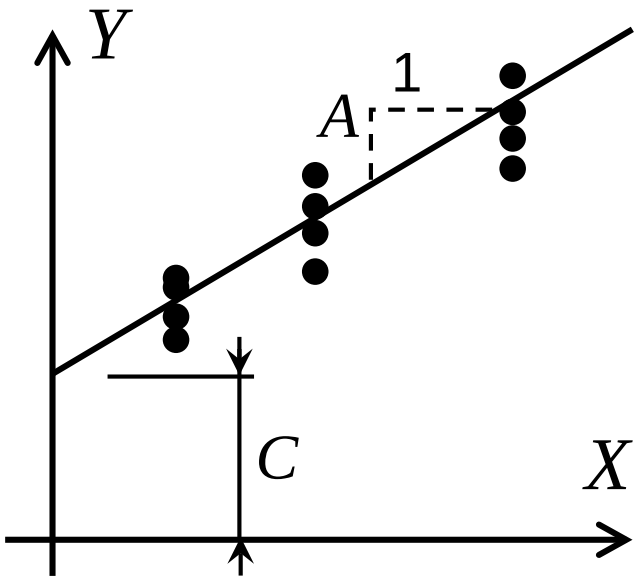
Asymptotic Matching yields the Size Effect Law (SEL) (1984, 1990, 2004)

$$\sigma_N = \frac{\sigma_0}{\sqrt{1 + D/D_0}}$$

In detail: ZP Bažant (2004), PNAS, p. 13400

ZP Bažant, Z.P. (1984). J. Eng. Mech. 110, 518—535; ZP Bažant, MT Kazemi (1990). IJF, 44, 111--131.

Size Effect Method to Measure Fracture Energy G_f and Material Characteristic Length c_f



$$\sigma_N = B f_t' \left(1 + \frac{D}{D_0}\right)^{-1/2}$$

$$= \sqrt{\frac{E' G_f}{g'(\alpha_0) c_f + g(\alpha_0) D}}$$

$g(\alpha)$ = dimensionless energy release function of LEFM

Transformation to linear regression:

$$Y = AX + C, \quad \text{with} \quad X = D, Y = \frac{E'}{g_0 \sigma_N^2}$$

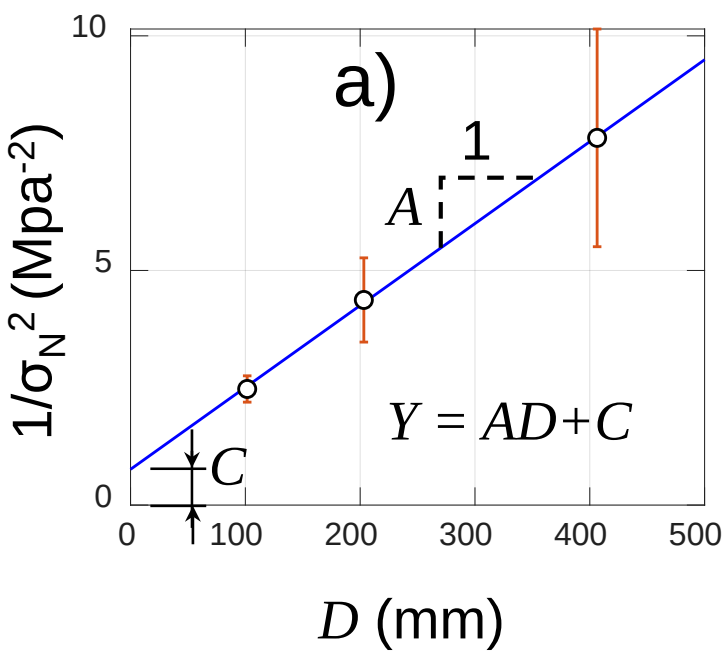
$$G_f = \frac{1}{A}, c_f = \frac{g_0}{g_0'} C$$

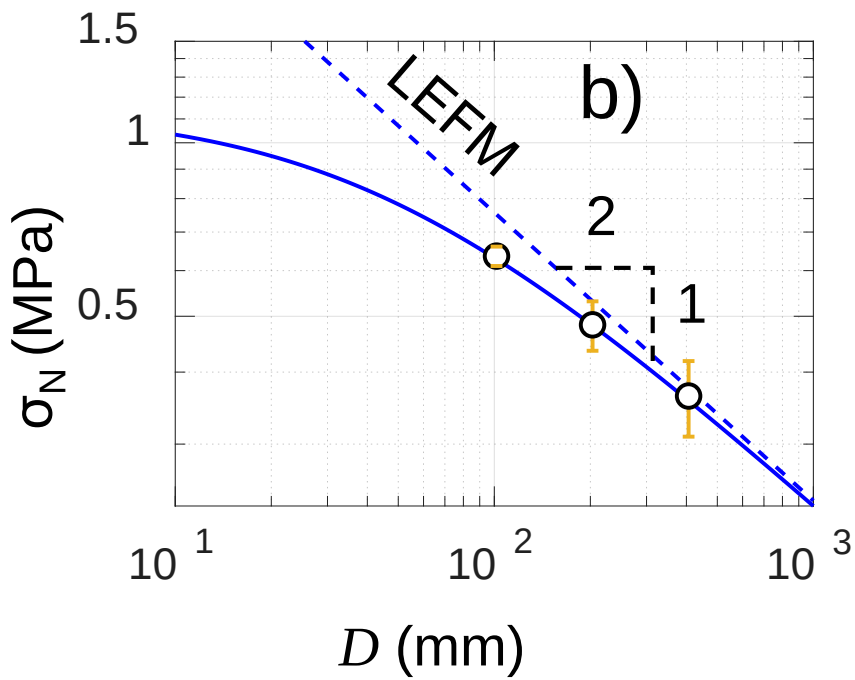
RILEM Standard Recommendation 1990
ACI-446 Recommendation

ZP Bažant, MT Kazemi (1990). IJF, 44, 111--131.

Size effect regression of 9 Gap Tests

to get Gf for, e.g., the medium crack-parallel compression (the means and the scatter width)





Note the closeness of fit

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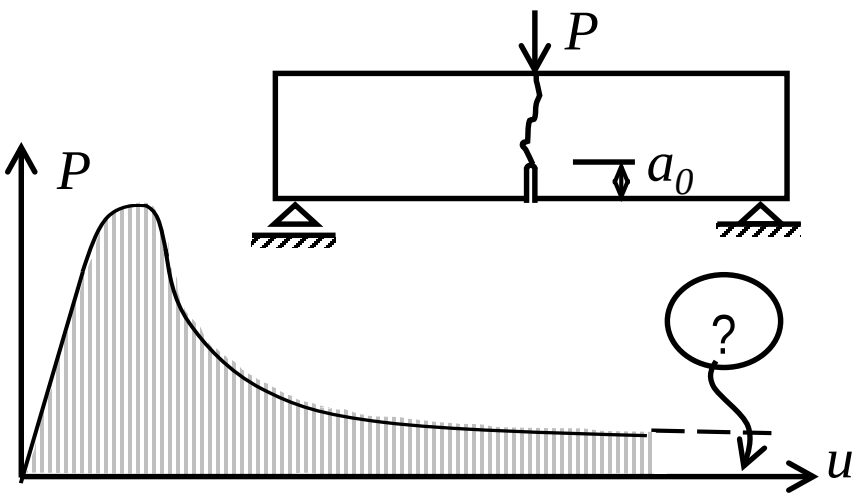
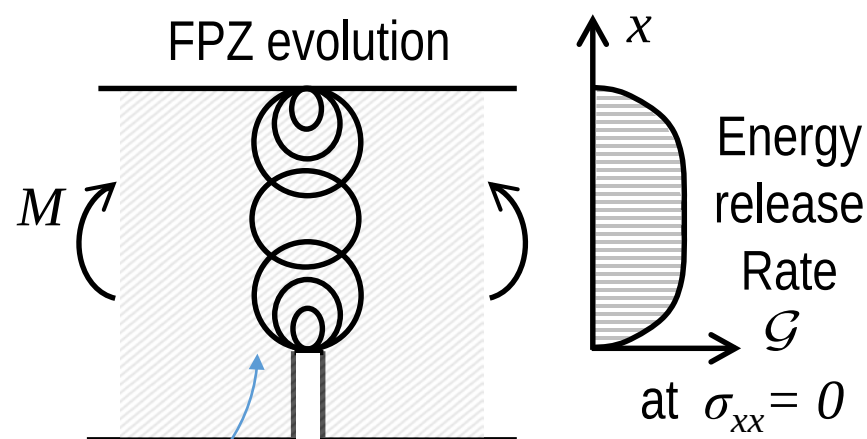
Classical Work-of-Fracture Method of Measuring Total Fracture Energy G_F





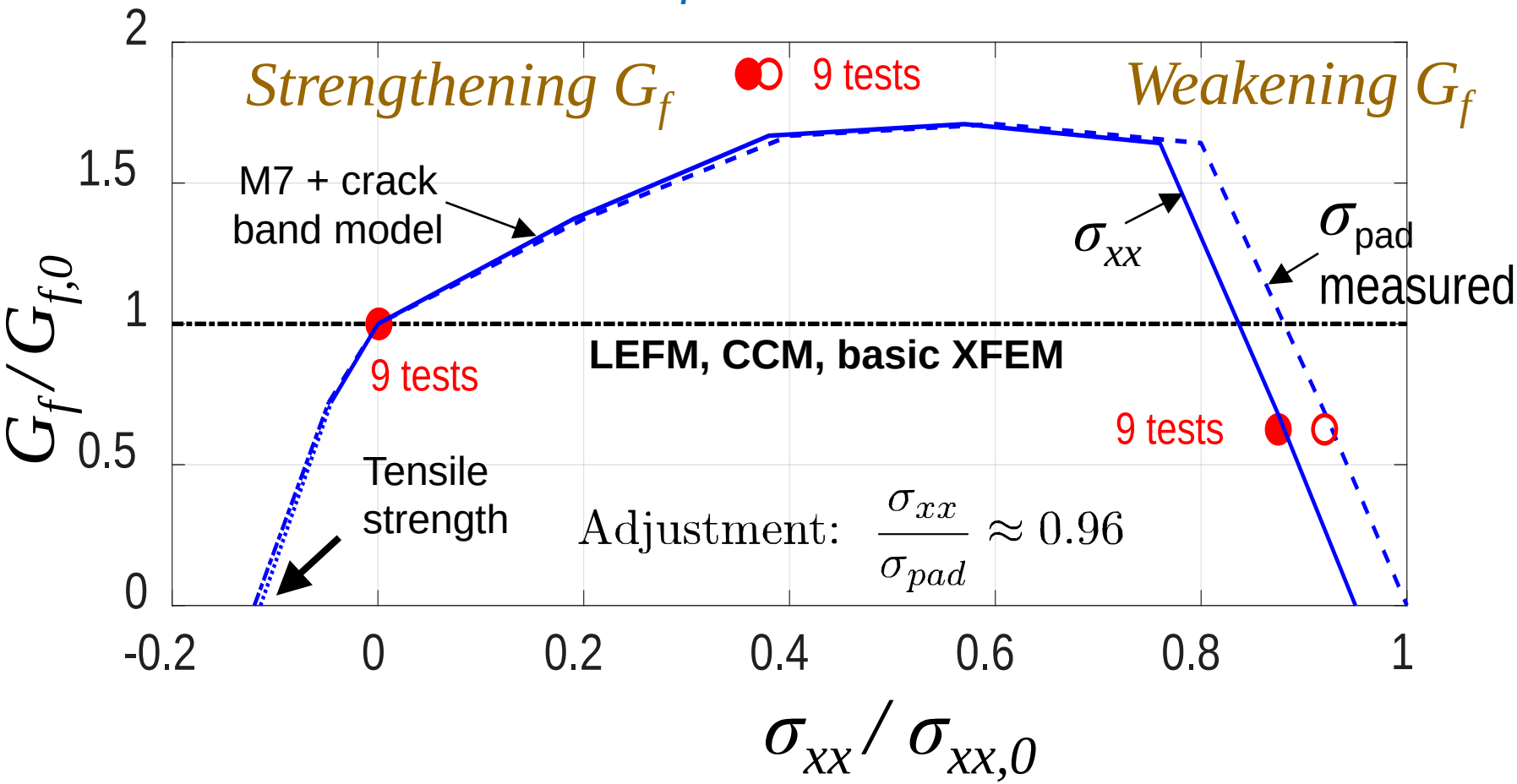
Nakayama 1965
Hillerborg 1976

Why was it not used?

1. J-integral varies with crack length x
2. Ambiguity of tail—ratios G_F/G_f, f_t/f_1 matter but vary
3. Tail hard to measure; stable postpeak required

III.
Gap Test Results and Mesoscale Physical Mechanism

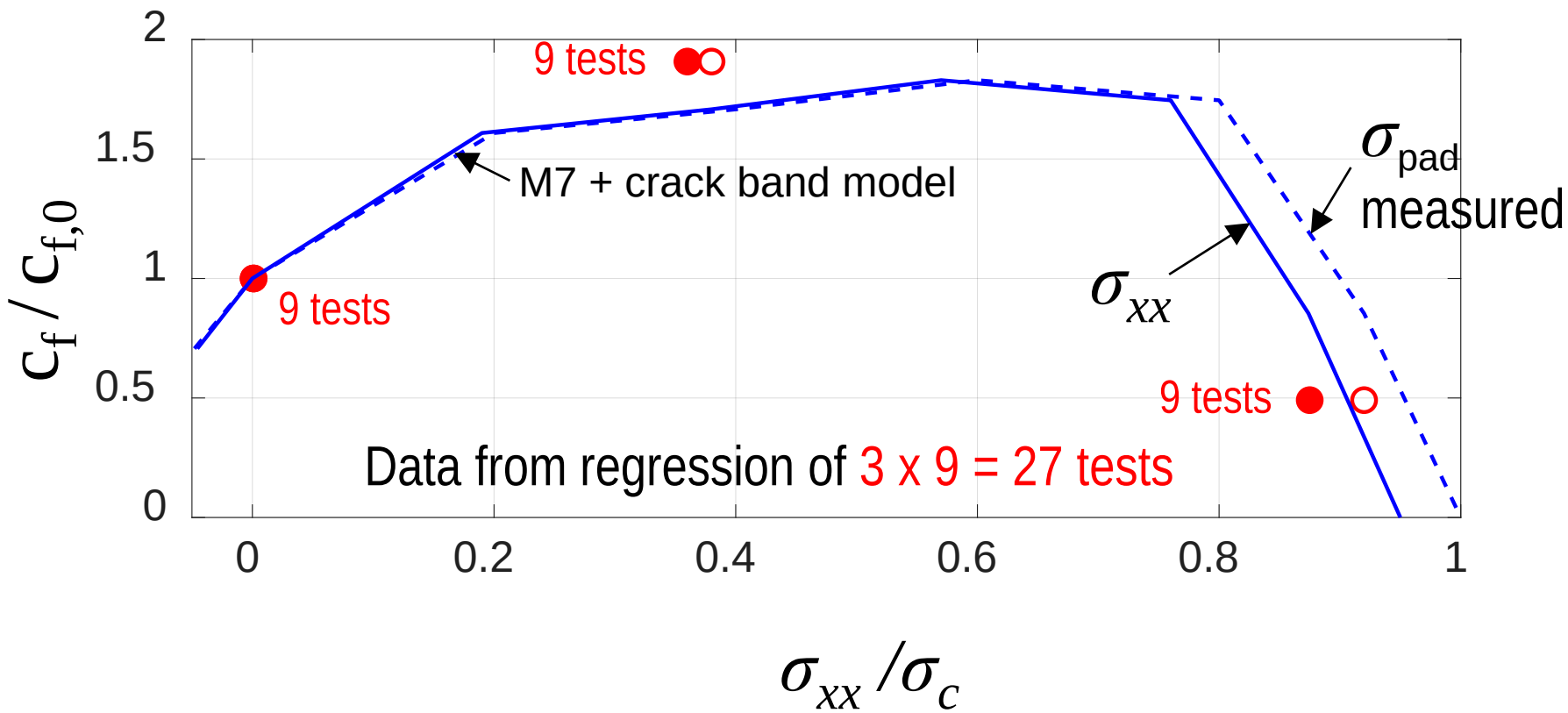
Found: Effect of crack parallel stress σ_{xx} on fracture energy G_f of concrete is strong



3 data points – based on regression of 3 x 9 = 27 experiments

M7 microplane prediction is calibrated by only f_c and $G_{f,0}$

Material characteristic length c_f dependence on crack-parallel stress σ_{xx}



Note: Approx. $c_f = 0.4$ FPZ length

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Meso-Scale Mechanisms of σ_{xx} Effect on G_f

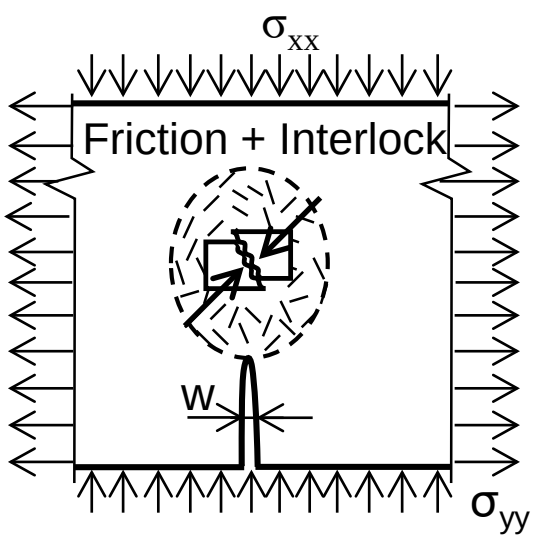
Regimes of:

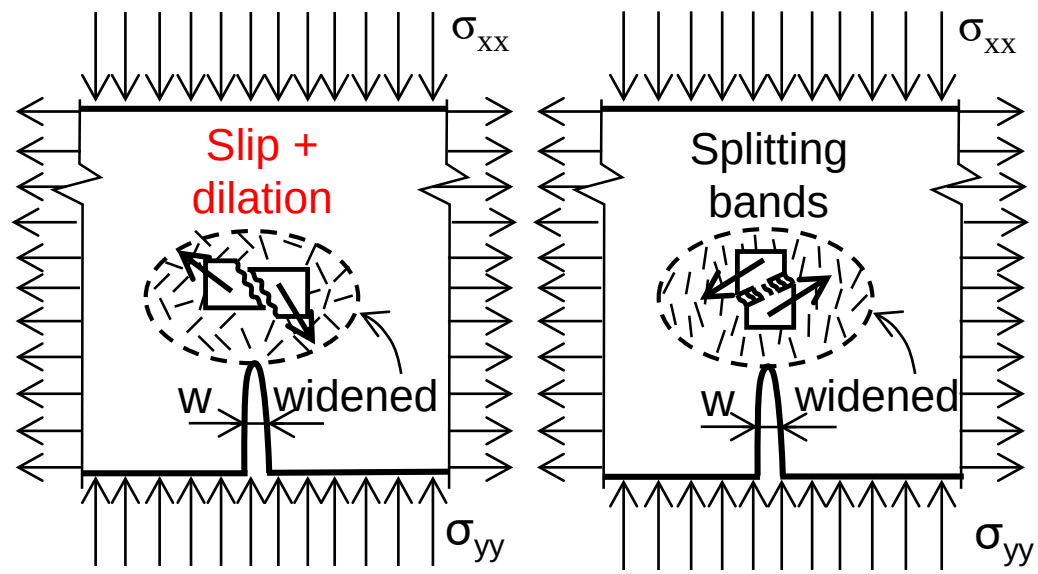
1) strengthening G_f

$0 < \sigma_{xx} < 0.75\sigma_c$

2) weakening G_f

$0.75\sigma_c < \sigma_{xx} < \sigma_c$





Static friction without slip enhanced by compression

Transverse <u>widening</u> of FPZ caused by inclined slips and splitting

IV.
FE Extrapolations and Predictions for Fiber Concrete and Shale

Extrapolations Based on Microplane Model M7 Approximately Validated by Gap Tests

The crack band model M7 with its default parameters was scaled to fit only:

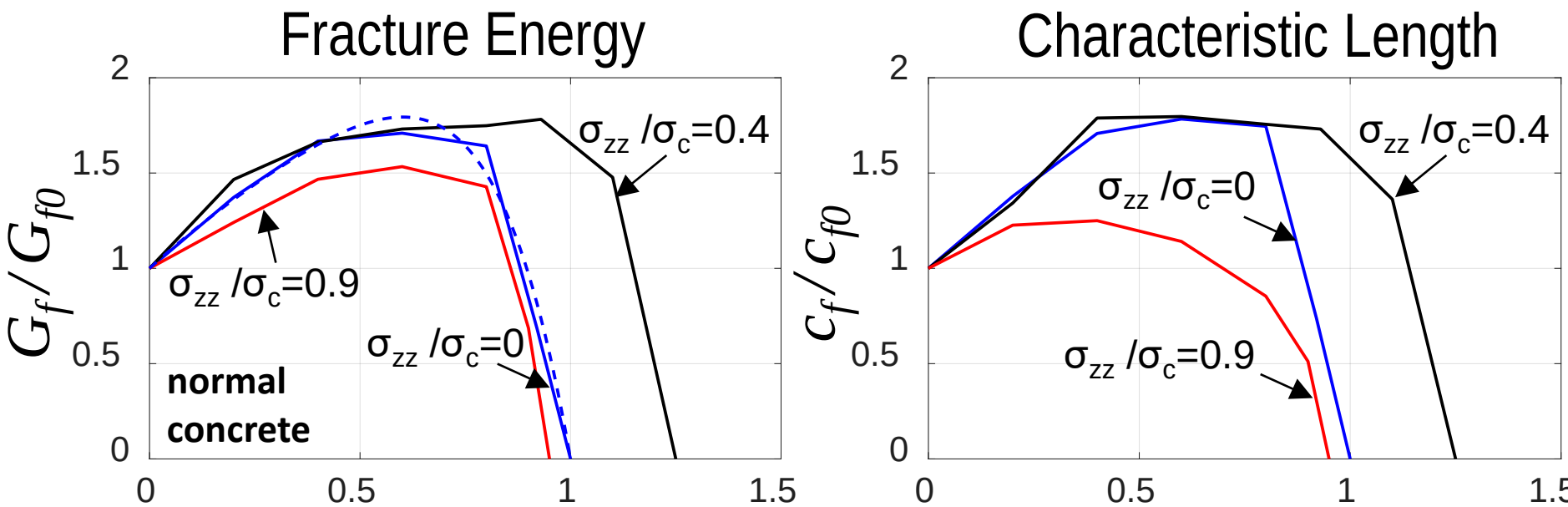
1) uniaxial compression strength of the concrete, and
2) the measured basic fracture energy (at $\sigma_{xx} = 0$).

The predictions for the 2nd and 3rd data points were acceptable (10% error) and excellent (1% error).
Overall 7% error (which is normal for concrete)

So it makes sense to use microplane model M7 to make predictions for **other situations and materials**

Effect of combined in-plane and out-of-plane stresses σ_{xx} and σ_{zz} on G_f and c_f

Calculated for Gap Test Geometry



σ_{xx} / σ_c

crack–parallel stress

Note the non-monotonic effect of σ_{zz}

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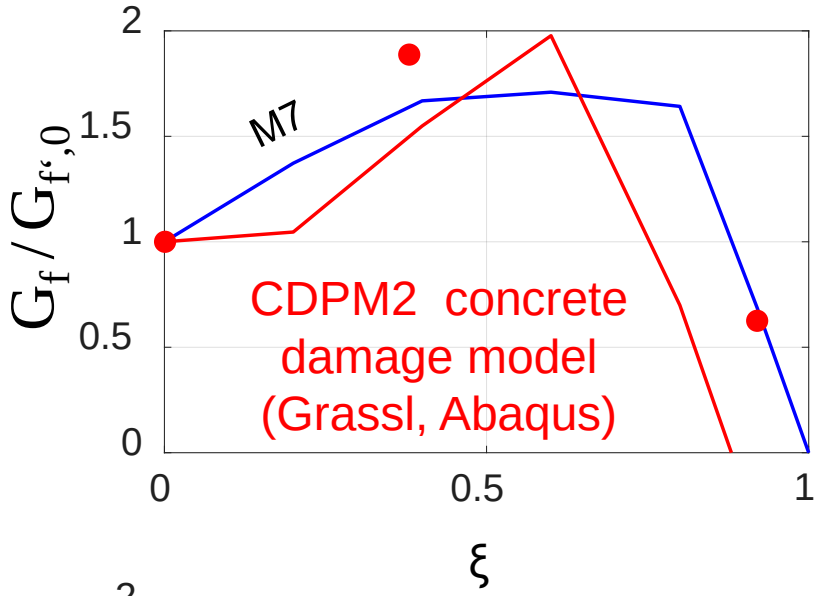
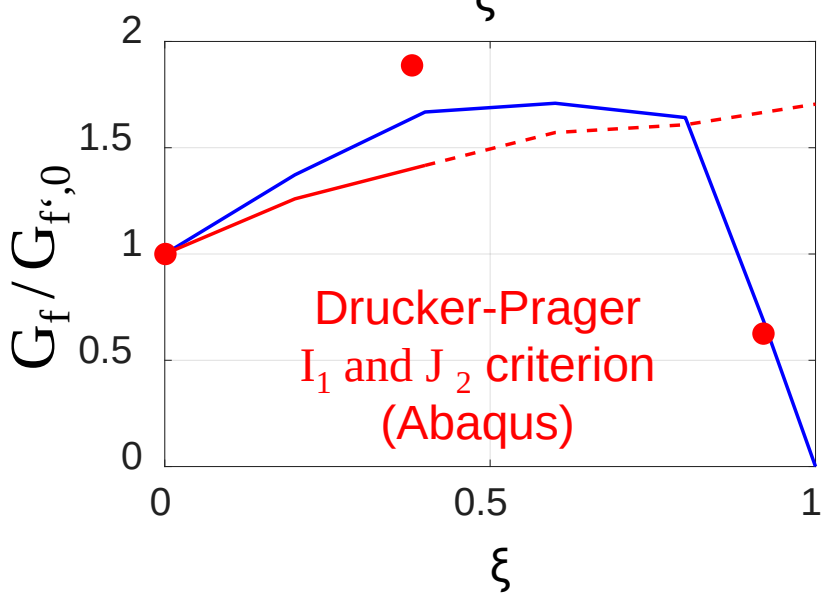
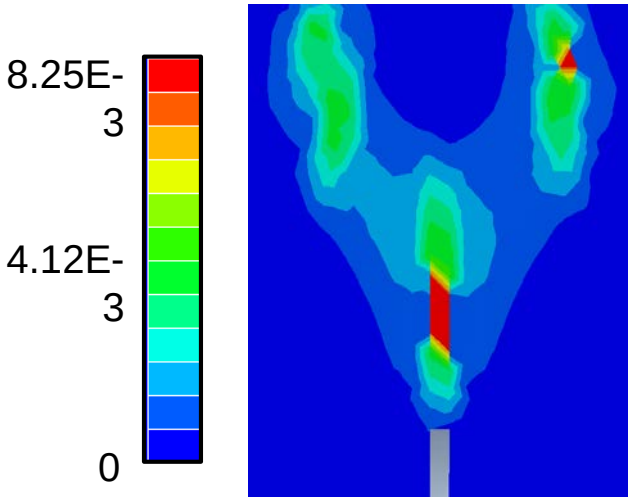
Comparisons with classical tensorial strength or damage models based on invariants

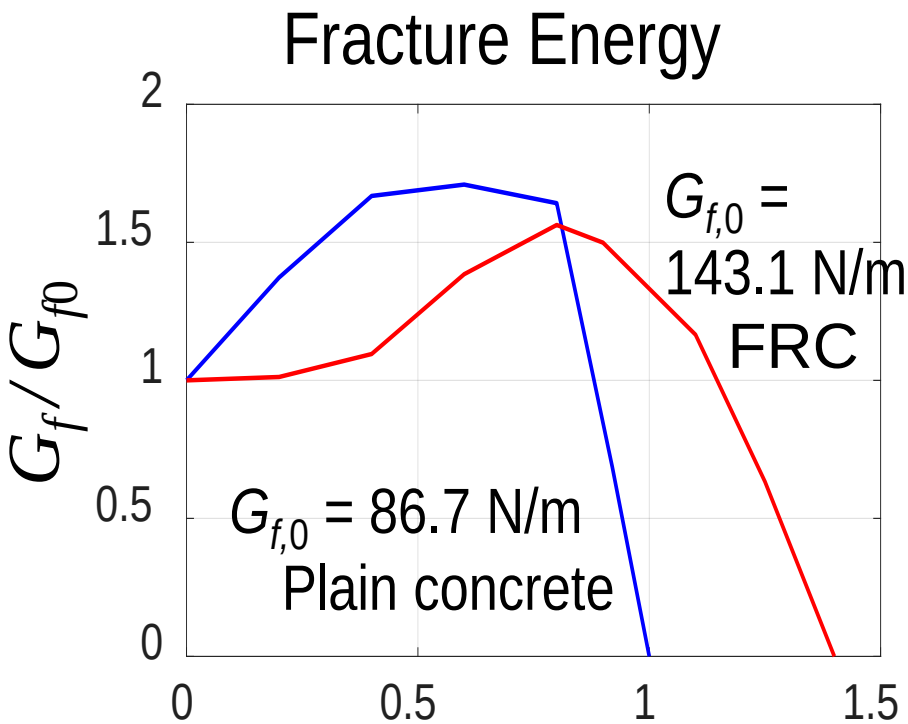


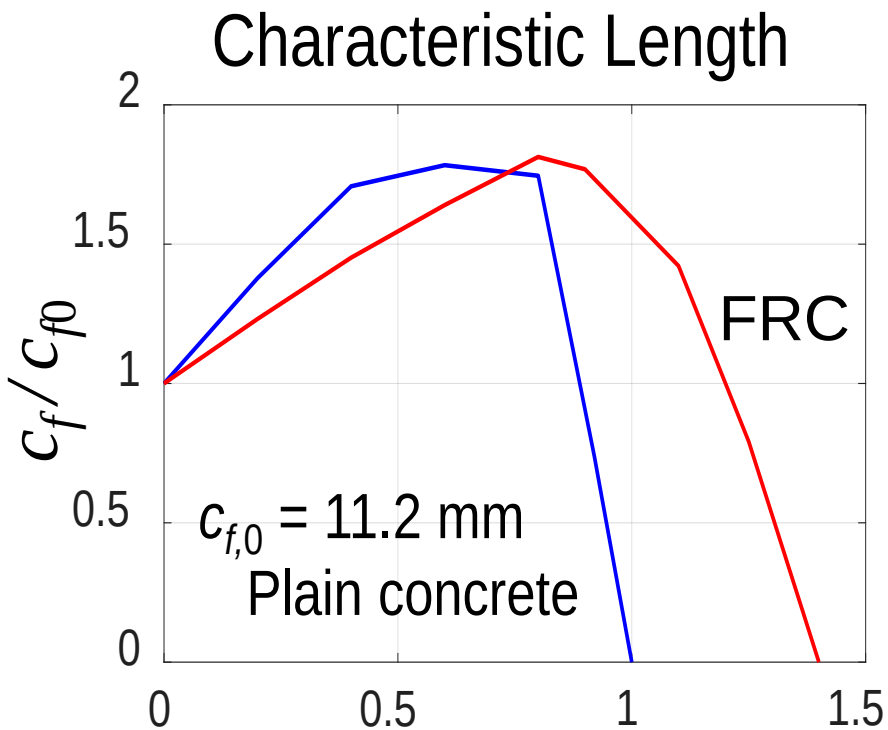




H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press

Fiber Reinforced Concrete (FRC) with 3% Dramix Steel Fibers vs. Plain Concrete





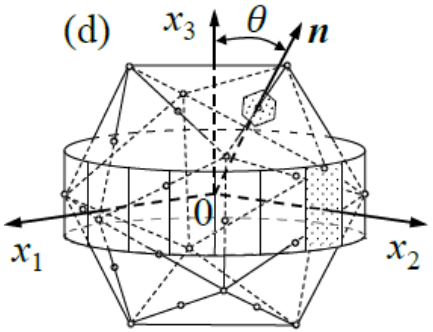
Crack–parallel compression $\sigma_{xx} / \sigma_{xx,c}$

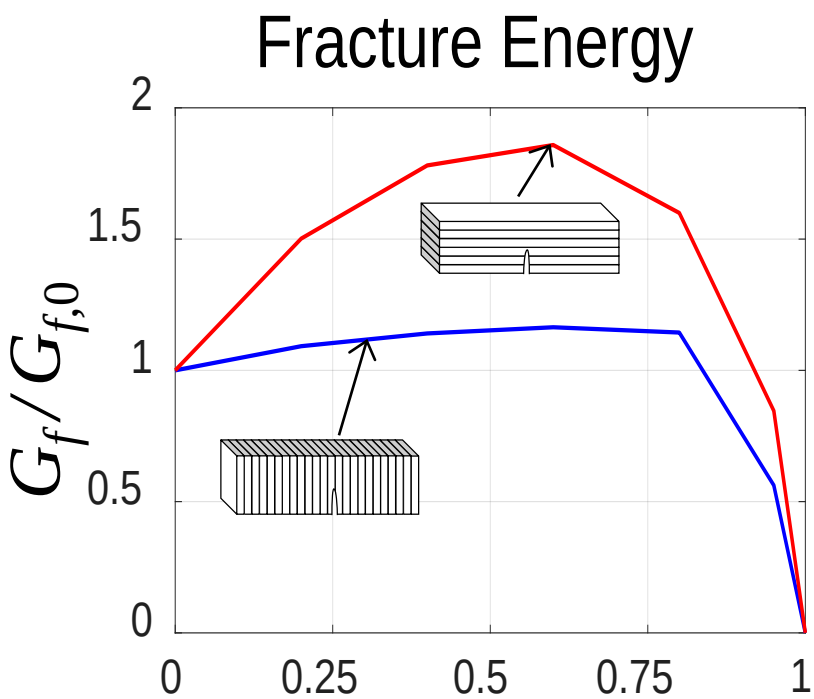
Calculated with M7 for concrete and M7f for FRC

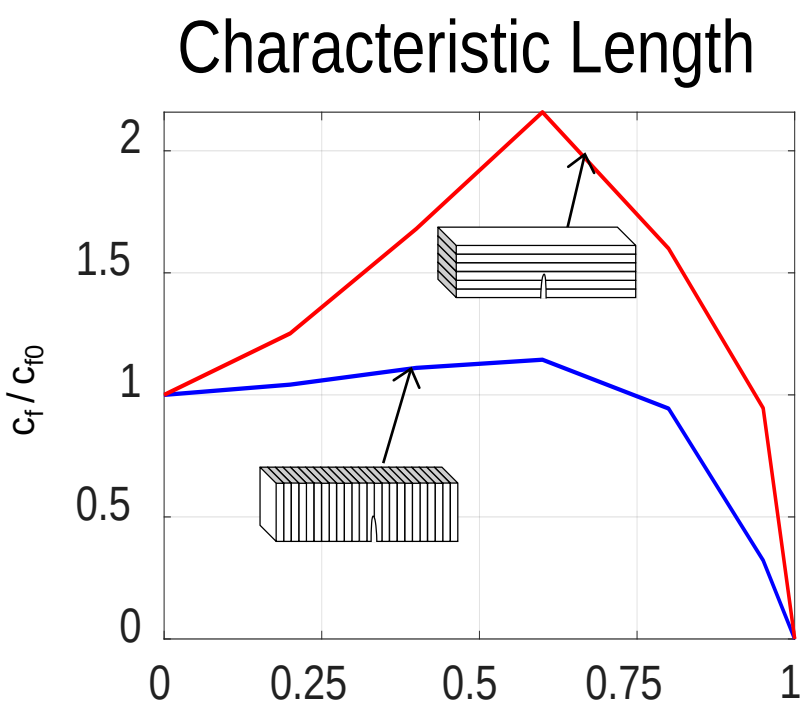
– tests yet to be made!

H. Nguyen, …Z.P. Bažant, JAM, in press; FC Caner, ZP Bažant (2013), Eng. Frac. Mech.105, 41--57

Gap Test for Gas Shale Predicted with Spherocylindrical* Anisotropic Microplane Model







Crack–parallel compression $\sigma_{xx} / \sigma_{xx,c}$

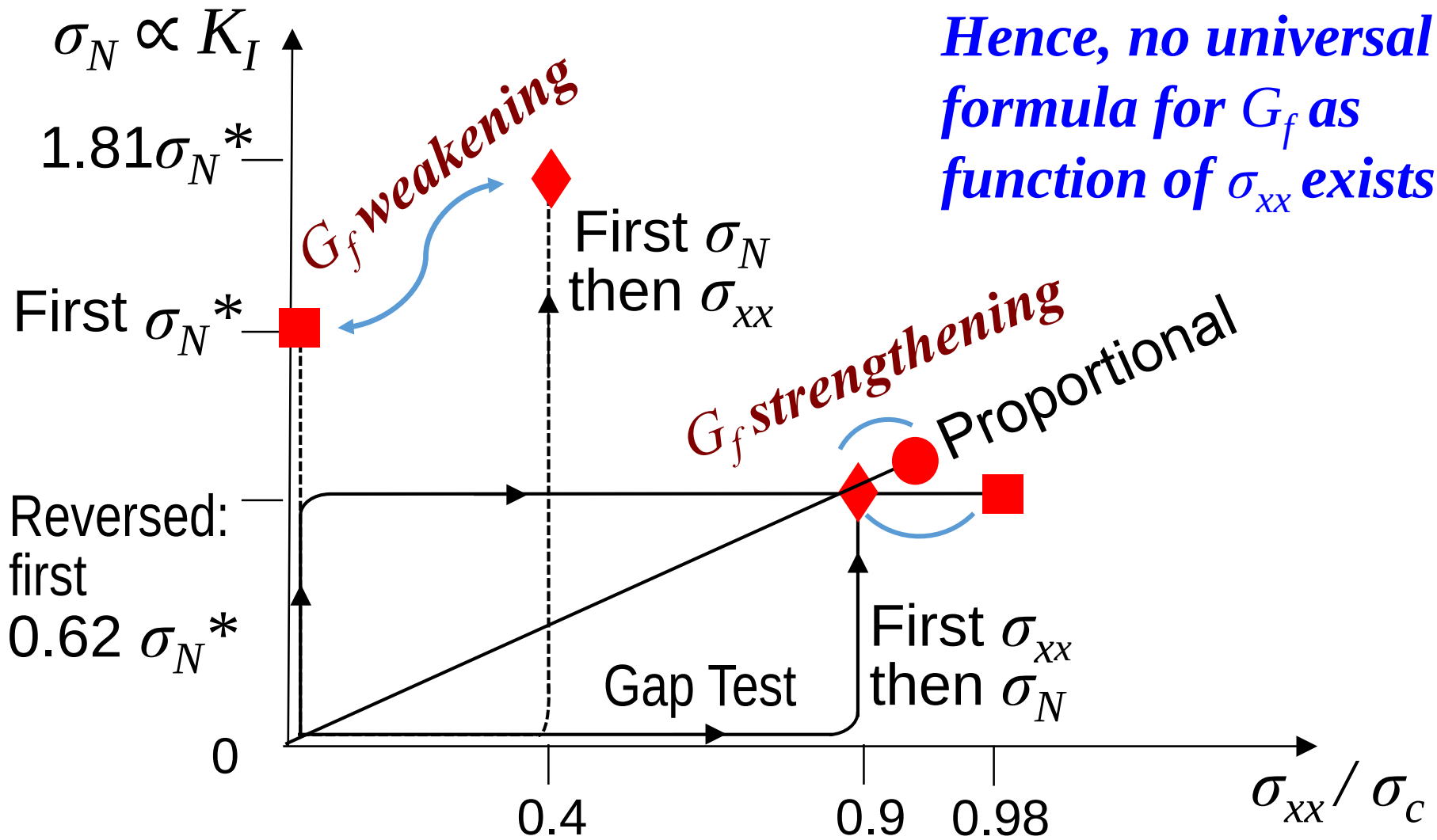
*Li, Cunbao,...Bažant, JMPS 103, 155-178 (2017)

– tests yet to be made!

H. Nguyen, M. Pathirage, G. Cusatis, Z.P. Bažant, JAM, in press

NOTE: After the Virus, collaborative tests will resume with LANL (Luke Fray, Hari Viswanathan, Bill Carey and Esteban Rougier)

Strong effect of history (or path) of loading, calculated for concrete



Hence, no universal formula for G_f as function of σ_{xx} exists!

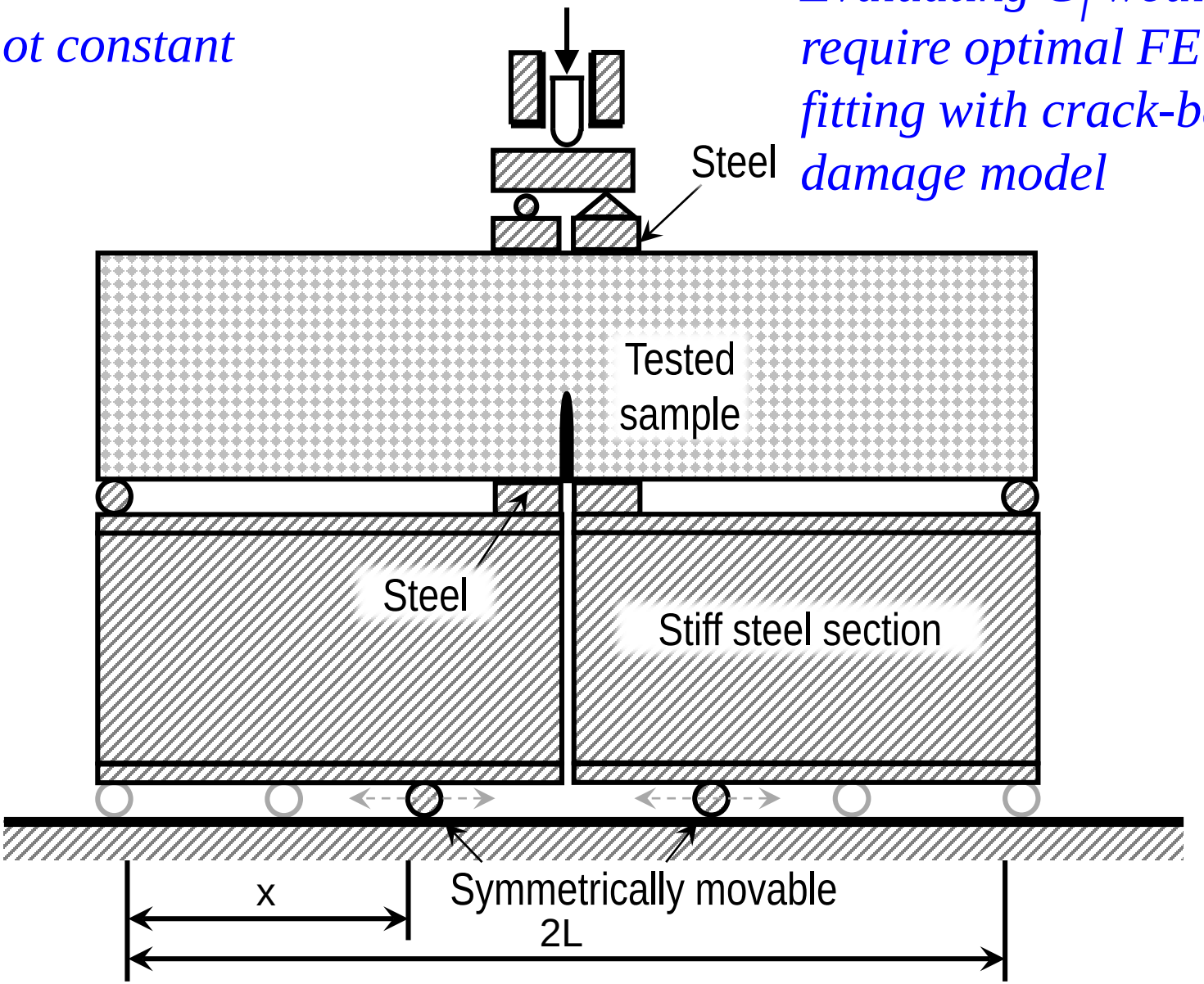
H. Nguyen,…. Z.P. Bažant, PNAS, in press; JAM July 2020

V.
Alternative Tests Considered and Retrospective

Alternative setup – proportional loading

Problem:
σ_{xx} is not constant

Evaluating G_f would require optimal FE fitting with crack-band damage model

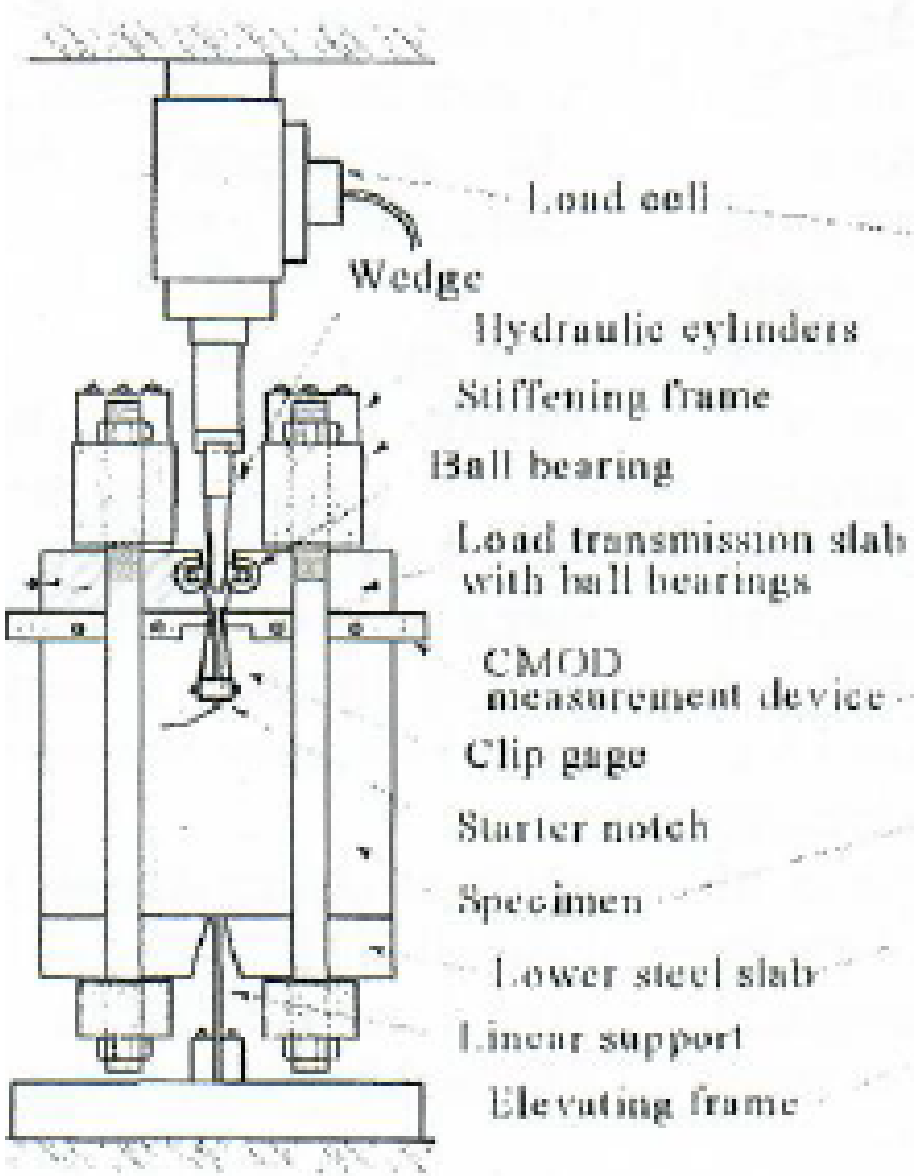


Tschegg's 1995 wedge-splitting test with crack-parallel compression

— pioneering idea!*

Tests showed some effect but were **ambiguous** because:

- <u>work-of-fracture method</u> was used
- <u>no</u> size effect was tested
- <u>non-uniform</u> stress field
- bending and friction from weight of hydraulic jacks



Ignored by fracture community due to lack of simplicity and high scatter

Retrospective

- Elasto-plastic metals: Stress triaxiality in an annulus around crack tip, led to extra parameter *Q* accounting for compressive T-stress (= σ_{xx}), and an increase of J-integral based on HRR field for ductile fracture (Shih, Hancock, Tvergaard,… 1990s).

- Crack path deflection due to T-stress in LEFM was solved by Cotterell and Rice (1980)

But both problems are different.

VI.

New Perspective of Fracture Mechanics of Quasibrittle Materials

Formula Approximating Present Gap Tests and FE Simulations

$$\frac{G_f}{G_{f0}} = 1 + \frac{a}{1 + b/\xi} - \frac{1 + a + b}{1 + b}\,\xi^s \qquad \xi = \frac{\sigma_{xx}}{\sigma_{xx,c}}$$

Can the existing FE programs for <u>LEFM, CCM, XFEM and phase-field model</u> be used? Hardly, because:

— Path dependence of damage is even stronger than it is in plasticity

— Different formulae would be needed for different materials, stress ratios $\sigma_{zz} / \sigma_{xx}$, plane strain, …

H. Nguyen, …Z.P. Bažant, JAM, in press. Prelim.: PNAS in press.

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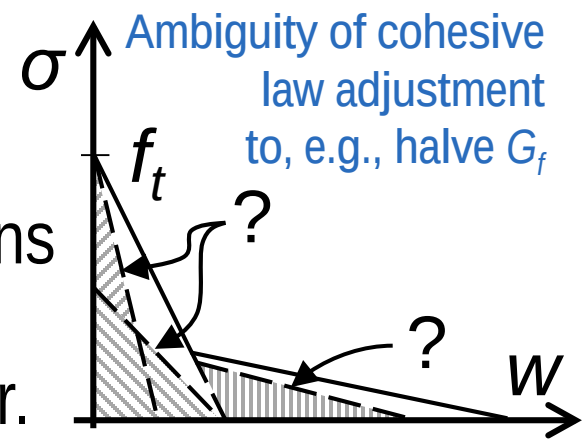
- To adjust G_f of the cohesive crack model is **ambiguous** since many different combinations of tensile strength f_t and softening slope are possible, and adjusting ratio G_F/G_f is unclear.



- Expected: A strong σ_{xx} effect in **anisotropic fiber-polymer composites** – e.g., a pressurized **fuselage** is under high **biaxial tension**.

- **Delamination** fracture is sure to depend on σ_{xx} , σ_{zz} .

Paramount Problem: Realistic Tensorial Damage Model

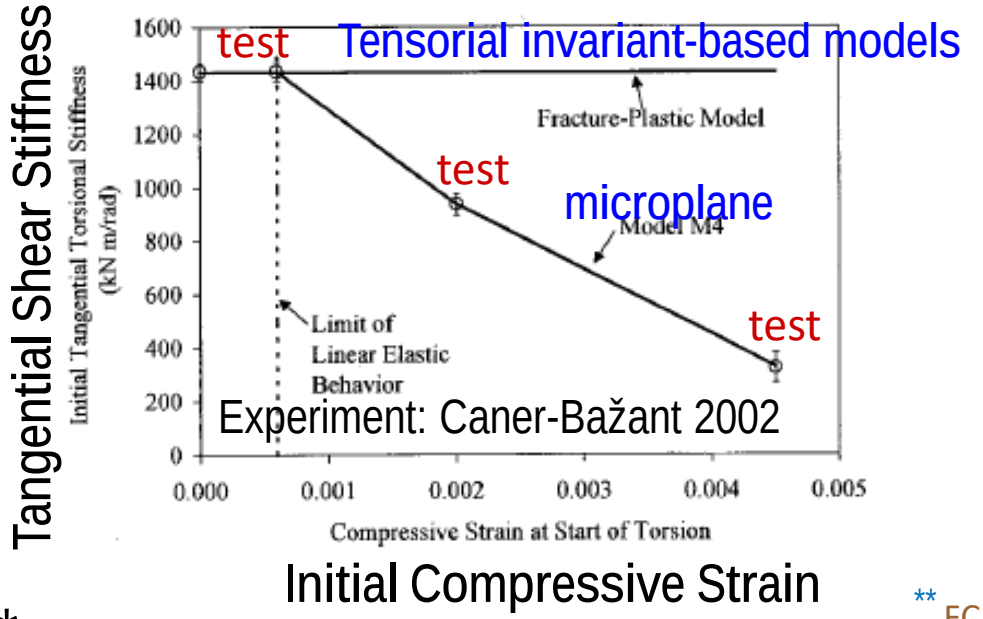
for concrete, fit 22 different benchmark triaxial tests (microplane model does)*

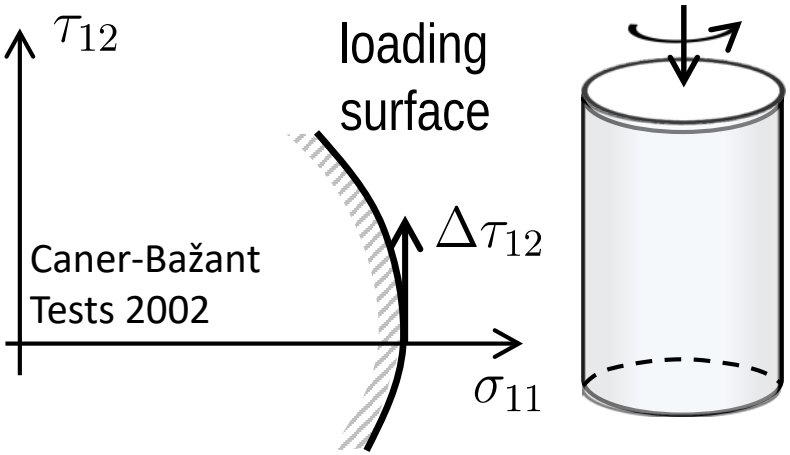
- *uni-, bi-, tri-axial* ▪ *splitting strength*
- *proportional or nonproportional*
- *post-peak softening at FPZ scale*
- *"**non-associated**" dilatancy of frictional slip (captured by M7)*
- *tension-compression-torsion*
- *unloading-reloading* ▪ *cycles…*

- *Plus **Vertex Effect**:*

– *Tangential stiffness at start of torsion vs. axial strain of compressed cylinder*

rotating principal stress axes

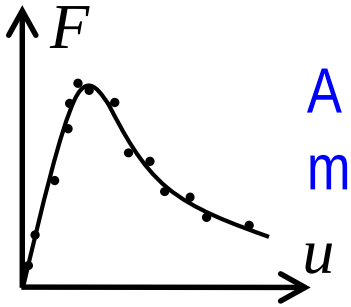




* FC Caner, ZP Bažant (2013), JEM, p. 1714

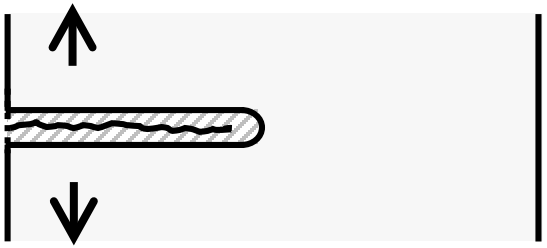
** FC Caner, ZP Bažant, J Cervenka, J. (2002). JEM, 24-33; ZP Bažant (!982), J Eng Mech ASCE 109, 849--865

Insufficient Verifications by Limited Test Data



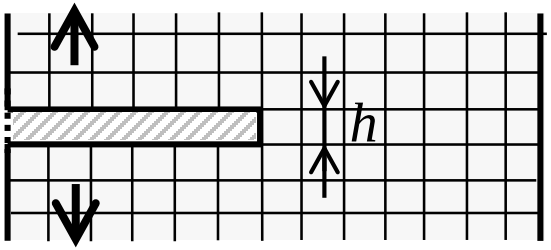
A hundred different damage constitutive models for concrete exist !

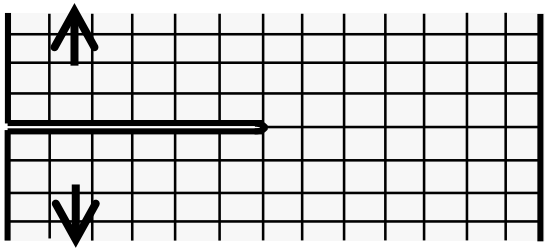
- All can work for <u>one crack</u>:



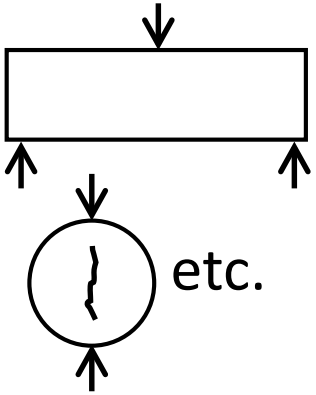
- All can fit Mode I test of $F(u)$ <u>softening curve</u>,

using:
- scalar softening $\sigma_{yy} = f(w)$
- I_1^{ε} volumetric softening
- J_2^{ε} deviatoric softening (~von Mises)
- Mazars' $\|\epsilon^+\|$ one-param. softening, etc.

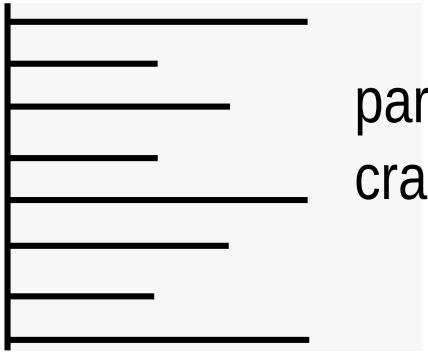




But should also fit:



- basic material test data and benchmark triaxial tests (with approx. FPZ size)
- multiple interacting cracks
- gap tests



parallel cracks

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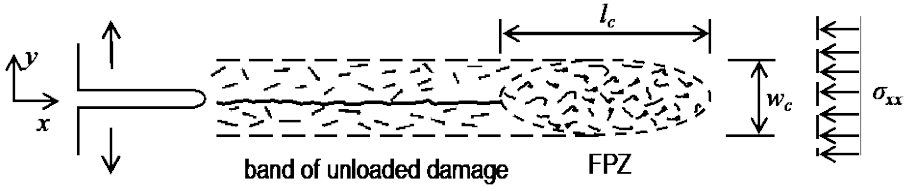
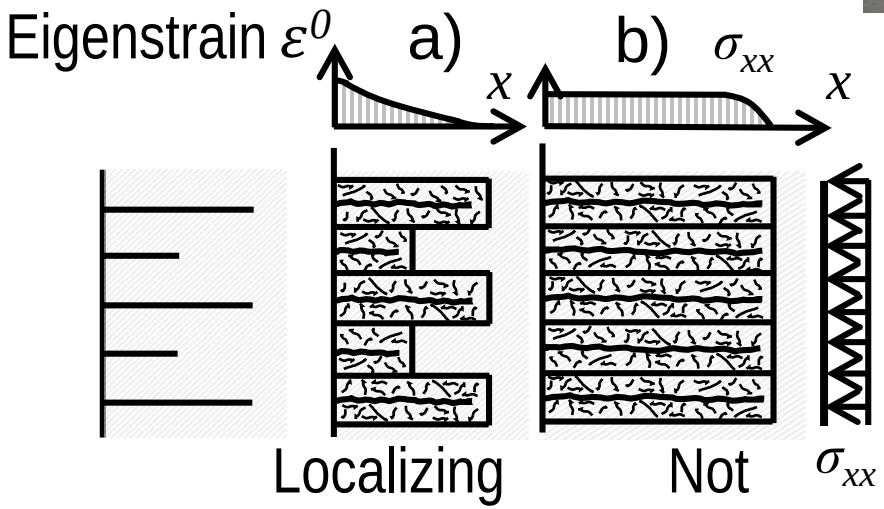
Both the tensorial nature of the FPZ and the minimum possible spacing of parallel cracks can be effectively captured by the crack-band model.

ad b



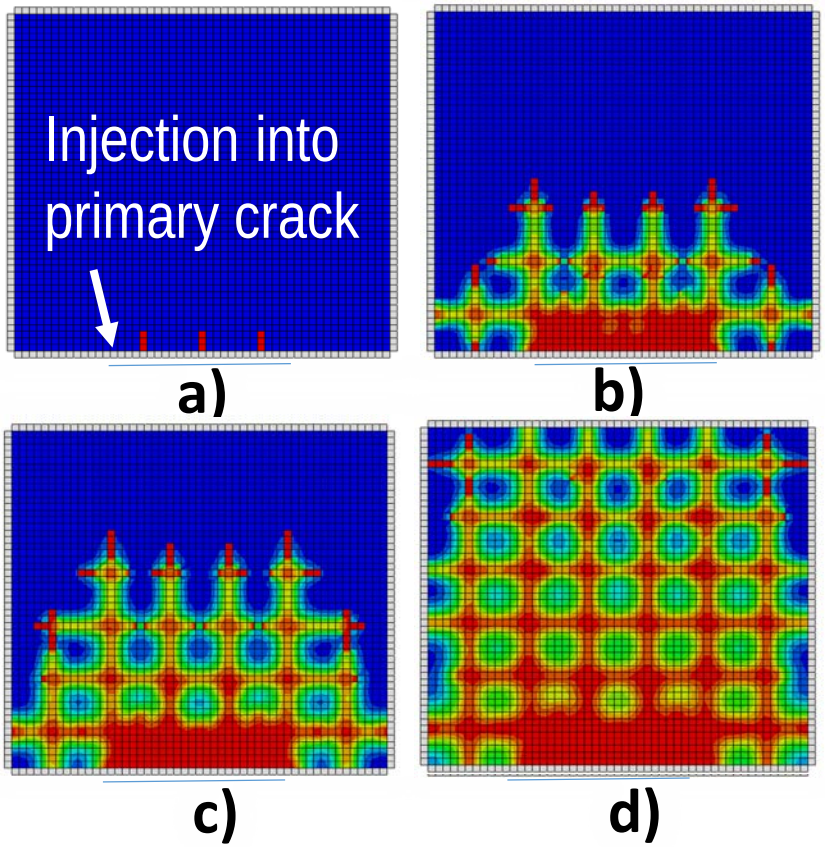


ad a

ZP Bažant, H Ohtsubo, Mech. Res. Comm. 4 (5), 353-366 (1977); Int. J. of Fracture 15, 443—456 (1979)

Effects of σ_{xx} , σ_{zz} in fracking of shale – probably great

3 km deep, vertical crack faces are under high <u>tectonic</u> and <u>overburden</u> crack-parallel stresses σ_{xx} , σ_{zz} , combined by body forces applied by diffusion pressure gradients.



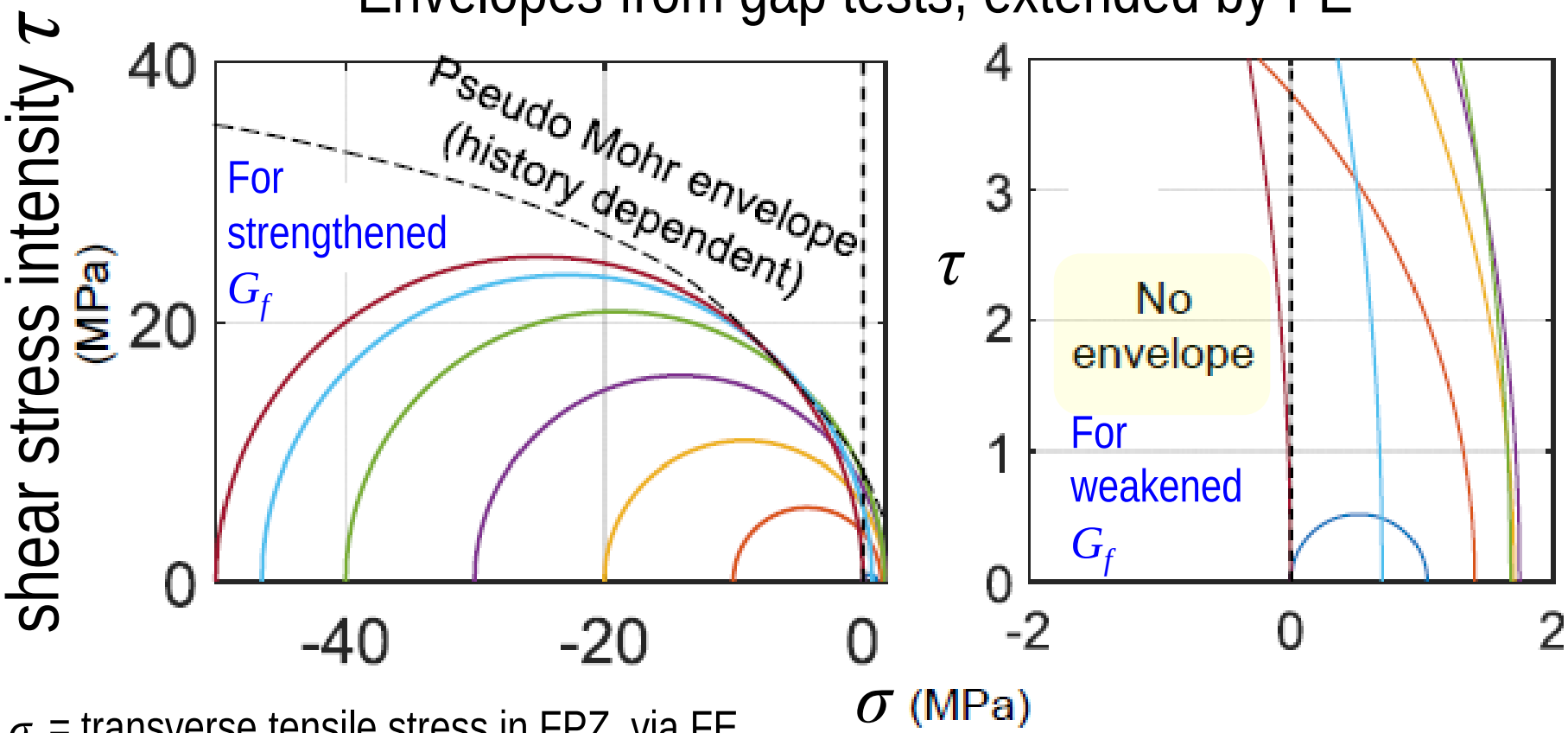
Simulated vertical hydraulic cracks in shale with <u>preexisting weak layers</u> <u>if G_f is constant</u> — no localization!

(Rahimi,…Bažant, PNAS 2019, p. 1532)

Continue using Mohr failure envelope for large crack-parallel stress σ_{xx}? — in doubt

—widely used in geophysics and in fracking studies.

Envelopes from gap tests, extended by FE



σ_I = transverse tensile stress in FPZ, via FE

$\sigma_{III} = \sigma_{xx}$ = crack-parallel compressive stress at FPZ

H. Nguyen,…. Z.P. Bažant, PNAS, in press

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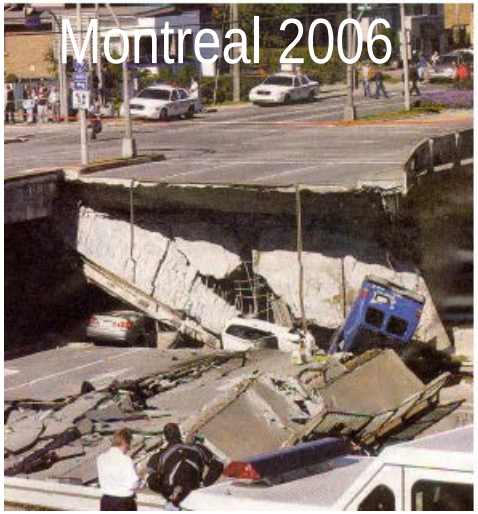
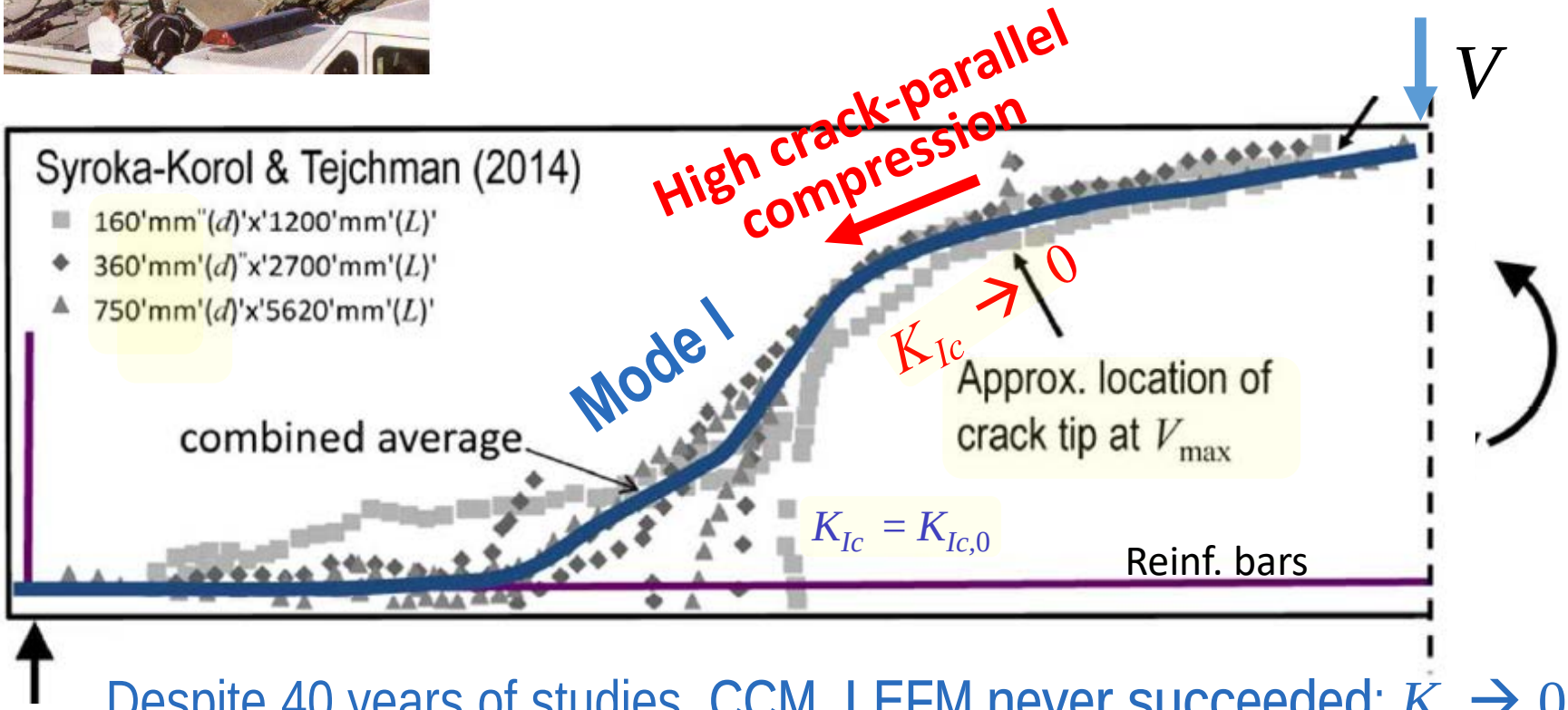


Gap Test proves that shear fracture of beams and slabs must be analyzed taking into account a large tensorial FPZ



Despite 40 years of studies, CCM, LEFM never succeeded; $K_I \to 0$ at V_{max} . *This was one experience that* *<u>motivated the gap test.</u>*

Q Yu, … Bažant, Z.P. (2016). Struct. Concr. 17 (5), 778-789; A Donmez, ZP Bažant (2019), Struct. Conc. 20, 1-13

Promise of Multiscale Approach

- Tensorial damage band shrunken into a line (Remmers, de Borst, Needleman, IJF 2013). Yields a cohesive crack with an embedded subscale tensorial damage band.

- But tensorial damage law remains a big question.

- Calibration by data fits not yet demonstrated.

- Conceptual problem: Minimum crack spacing will not be enforced automatically. Parallel cracks?

Promise of Phase-Field Model

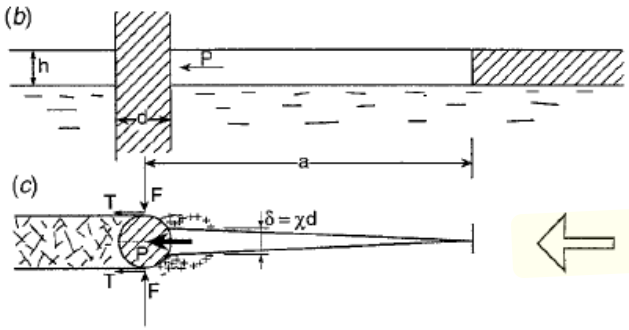
– *computationally enticing, alternative. But:*

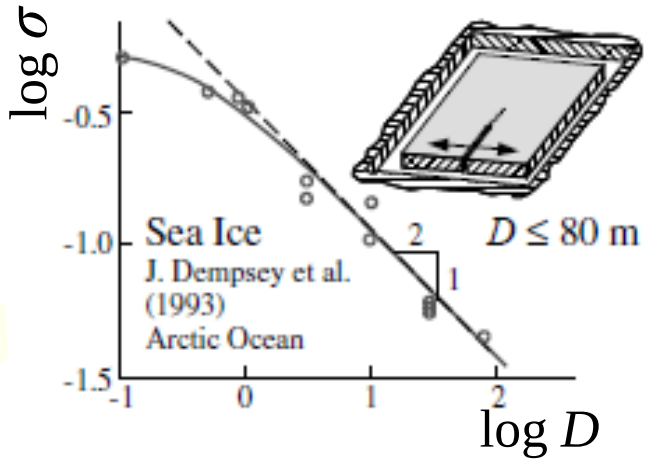
- Single parameter damage—inadequate.

- Needs a realistic tensorial model for softening damage, such as microplane, validated and calibrated by existing triaxial material tests of diverse types, esp. of FPZ size

- Transition from distributed damage to isolated bands? Minimum spacing of parallel cracks?

11 Fracture of Sea Ice with σ_{xx}





ONR Size effect tests 1993, on Arctic Ocean (Dempsey, Bažant collab.)

In 1980s, forces *F* measured on legs of oil platforms in Prudhoe Bay were an <u>order-of-magnitude less</u> than FE predictions with buckling.

— <u>One (incomplete) explanation:</u> Size effect

— <u>Now 2nd explanation</u>: Crack-parallel compression in <u>large FPZ</u> <u>reduced</u> G_f (or K_{Ic}) <u>near 0</u>.

In glaciers — σ_{xx} in vertical cracks, depth >1 km

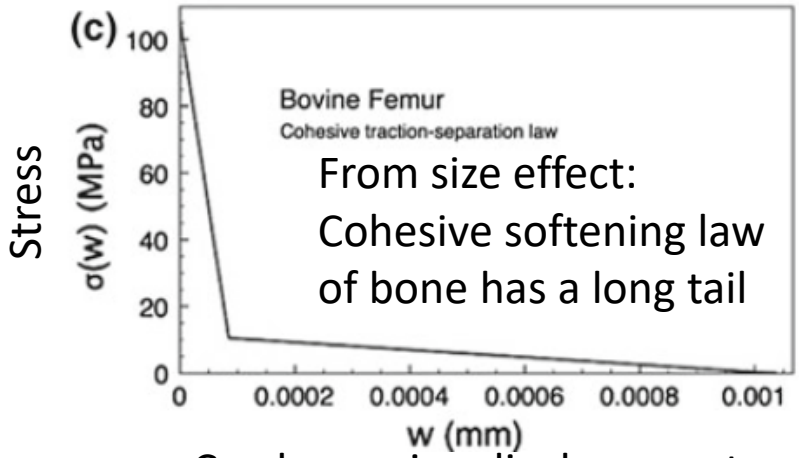
ZP Bažant (2002), J. Appl. Mech. ASME 69, 19--24.

Expected: Effect of σ_{xx} , σ_{zz} on fracture of bone and biomaterials

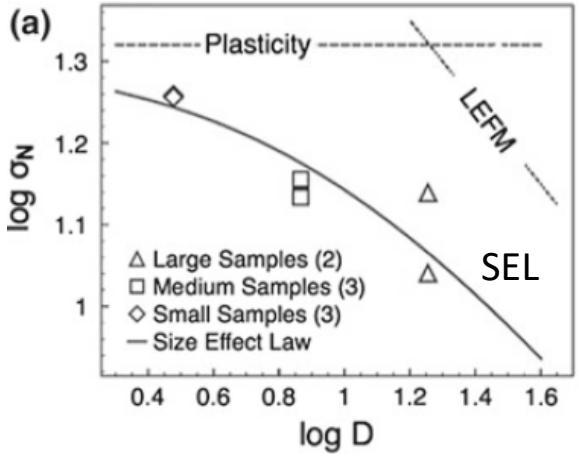
Why? Observed size effect implies large FPZ

Hip fracture, tooth fracture, …

often occurs at high σ_{xx}



Crack opening displacement



Bone tests by Bažant, Kim, Yu (IJF 2013)

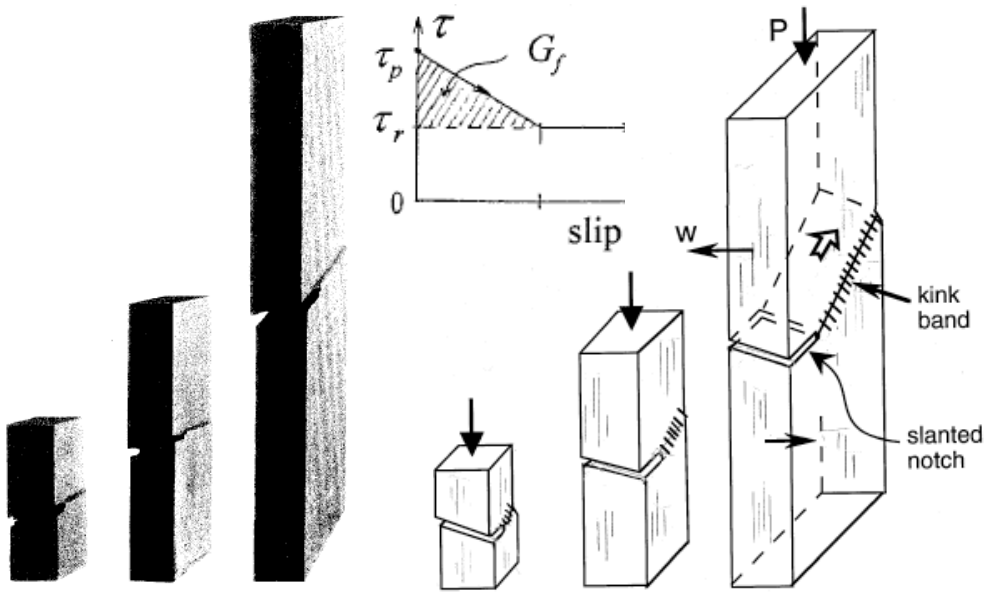
ZP Bažant, KT Kim, Q Yu, K. Xu, IJF 181, 2013

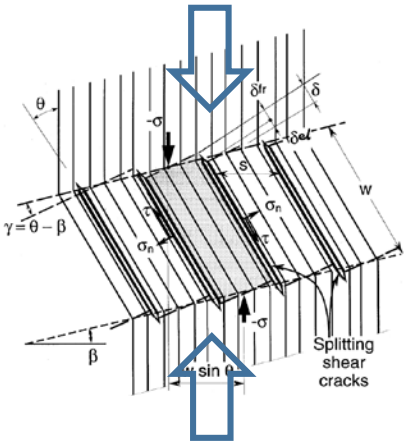
Tests of PEEK at Northwestern

(Bažant-Kim,…, IJF 1999)

- Kink-bands in fiber composites are a fracture propagation problem with strong size effect

- σ_{xx} effect — unknown but likely.





Kink band micro-buckling with shear fractures leads to size effect

ZP Bažant, J J-H Kim, … IJF 95, 103-141 (1999)

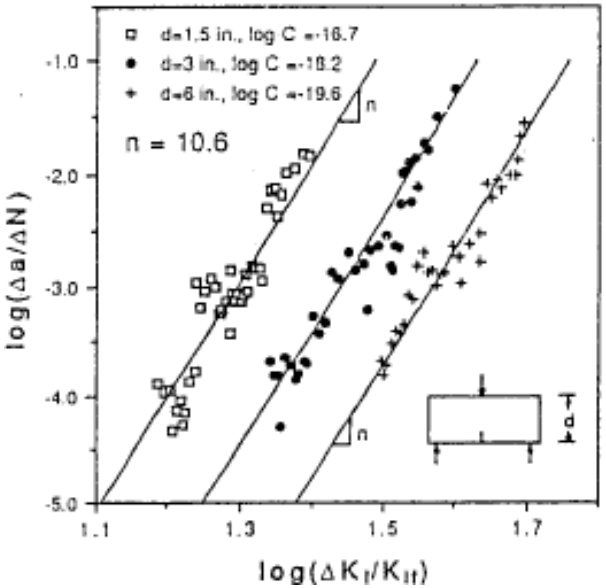
Large FPZ suggests:

effect of crack-parallel σ_{xx} ,σ_{zz} on subcritical crack growth

1) <u>Charles-Evans law</u> for static fatigue crack growth in rocks or ceramics, and

1) <u>Paris law</u> for cyclic fatigue crack growth in quasibrittle materials.

Bažant & Xu tests (ACI J 1991), for 3 specimen sizes



Size effect proves large FPZ

ZP Bažant, K. Xu, ACI Materials J 88 (4) 390—399 (1991).
ZP Bažant, PC Prat, M Tabbara, ACI Mat J 87, 12—19 (1990).

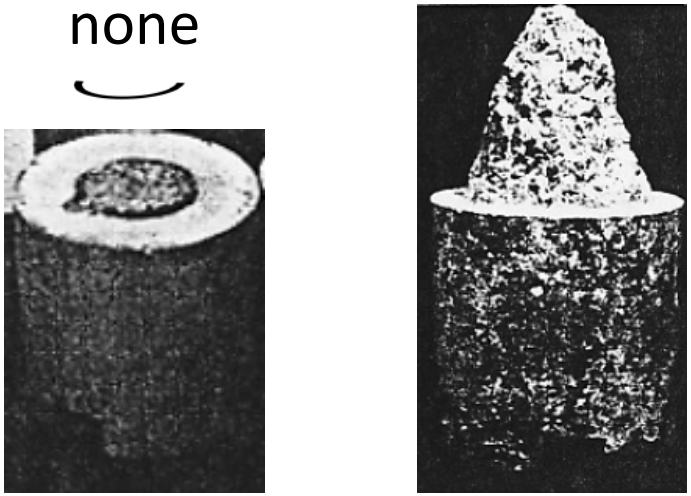
Tensorial Aspect of Shear Fracture

In concrete, compression changes a planar shear crack to an inclined conical shear crack.

Inexplicable if constant G_f or K_c govern.

With none

With axial compression



Torsional shear tests

Bažant, Prat, Tabbara, ACI J. 1990, 12-19

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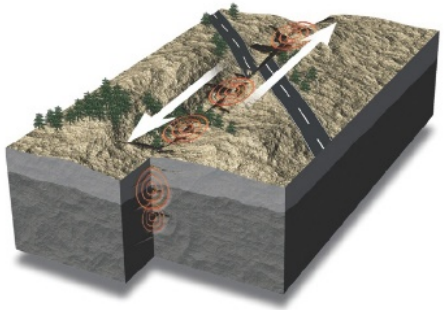

- Seismically sliding geological faults are very thin (< 1 mm), but if the FPZ at front is not, then σ_{xx},σ_{zz} should matter for their propagation.

- Cracks growing in prestressed concrete have a different G_f.

- The G_f variation is generally not expected in metals, fine-grained ceramics or polysilicon, since their FPZ is of micrometer scale. But it could matter for MEMS, or devices < 10^{-4} m.

Conclusions

- Gap Test – simple, unambiguous interpretation – one **statically determinate** system transits to another – size effect test of G_f possible

- FPZ – Tensorial !
  Effective simple approach – FE crack band model.

- The CCM and LEFM (and XFEM) are approximations of rather **limited applicability**.

- LEFM and CCM remain essential for understanding and teaching fracture mechanics, and for providing accurate benchmark solutions of special cases which tensorial damage mechanics must match.

- The evidence from the gap test looks compelling but so far is scant and deserves broad scrutiny. **Extensive testing and calibration** will be needed – for most quasibrittle materials.

Afterthought

Many hot research subjects become closed in a few decades. But, like turbulence, fracture mechanics is different.

This formidable subject has been researched for a century, and probably will for another century.

Thanks for listening!

Download from www.civil.northwestern.edu/people/Bažant video link: https://youtu.be/UyTeopPModA

612.pdf. H. Nguyen et al., Perspective of Fracture Mechanics Inspired by Gap Test with Crack-Parallel Compression. Proc. National Academy of Sciences 52 (2020) June, in press.

613.pdf H. Nguyen et al., Gap test of crack-parallel stress effect on quasibrittle fracture and its consequences. ASME J. of Applied Mechanics 87 (2020), 07, pp. 1012-1—11.